

# THE MONIST

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## PRAGMATISM AND MATHEMATICAL LOGIC.\*

IT is certainly not one of the least of the merits of the *Leonardo* that it has established lines of communication and encouraged the exchange of ideas between exponents of philosophical studies belonging to the most diverse and distant intellectual fields—between logicians and estheticians, between moralists and economists, mathematicians and mystics, biologists and poets.

Pending the possibility of a comparative examination of the results obtained, or prepared, by the development and exchange of ideas in all these various directions, it will not be irrelevant to summarize here in a schematic synopsis such of these results as relate to one of the most important lines of communication which the *Leonardo* has helped to construct and keep in operation, that is, the line which joins the various domains of Pragmatism with those occupied and cultivated by the "mathematical logicians." A significant indication of the intimate connection between these two fields of philosophical research may be deduced from the fact that the sponsor of the denomination and concept of "Pragmatism" (Ch. S. Peirce) is himself likewise the initiator and promoter of an original trend in logico-mathematical studies.

It is not, however, from the labors of the school of

\* Translated from the Italian by H. D. Austin. This paper appeared originally in the *Leonardo*, Feb., 1906.

Peirce, but rather from those of the Italian school, headed by Peano, that I purpose here to take my material for the determination of what might be called the "pragmatic characteristics" of the new logical theories.

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One point of contact between logic and pragmatism is found in their common tendency to regard the value, and even the meaning, of every assertion as being intimately related to the use which can be made, or which it may be desired to make, of it for the deduction and construction of particular consequences or groups of consequences. This tendency is manifest, among the mathematical logicians, especially in their revision of the criteria employed in the choice and determination of *postulates*, that is, in the choice of those propositions which in each separate branch of deductive science are to be admitted without demonstration. Instead of conceiving of the difference between postulates and the other propositions which are demonstrated by means of them as consisting in the possession on the part of the former of some special character which renders them *per se* more acceptable, more evident, less disputable, and so on; the mathematical logicians regard postulates as propositions *on a par with all the others*. The choice of such "postulates" may differ according to the *end* in view, and must, in any case, depend upon an examination of the relations of dependence or connection which may be established between these "postulates" and the remaining propositions of a given theory, and upon a comparison with the form into which the treatment as a whole would develop under conditions of varying choices. If the relations between postulates and dependent propositions might formerly have been likened to those which subsist, in a state under autocratic *régime*, between the monarch or the privileged class and the rest of the social body, the work of the mathematical logicians

has been somewhat similar to that of the inaugurators of a constitutional or democratic *régime*, under which the choice of the rulers depends, at least theoretically, upon their recognized ability to exercise temporarily determined functions, to the public welfare. Postulates have had to relinquish that species of "divine right" with which their pretended evidence seemed to invest them, and resign themselves to becoming, instead of the arbiters, the *servi servorum*—the employees simply—of the great "associations" of propositions which make up the various branches of mathematics. With this same tendency are connected also the requirements regarding the maximum reduction of their number and pregnancy, the exact determination of their applicabilities and spheres of validity, and so forth.

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A second resemblance, of not less importance, between the pragmatists and the mathematical logicians appears in their common repugnance to the vague, indefinite, generic; and in their care to analyze every assertion into its simplest terms, whether referring directly to *facts*, or to *connections between facts*. It is thus that both constituencies have come, each for itself and in its own way, to recognize the unreality of a great part of the distinctions which have been handed down from scholastic logic to the modern "theories of knowledge," and to subject others of these distinctions to critical analyses from which they have emerged in a sense transfigured, restored, enriched with new and more important significances.

Thus, by introducing the conception of "possible definition" a clear recognition has been reached of the wholly relative character of the distinction between the "essential properties" of a given figure or mathematical entity and the other properties possessed by it. In like manner the distinction between affirmative propositions and negative, and that between particular propositions and

general, have been absorbed in the one and more important distinction between propositions affirming the interdependence of two facts and propositions affirming the *possibility* or the "*non-absurdity*" of the coexistence of two or more facts. This involves, too, the invalidating of the distinction between categorical and hypothetical general propositions.

The recognition of the hypothetical nature of general propositions has also helped to turn attention to the "tacit restrictions," or unspecified limitations, upon which their validity depends. A good example of this is the observation of Maxwell (cited by Roiti in his *Elementi di Fisica*, 1894, p. 65): that even the simplest propositions as to areas, *e. g.*, that "the area of a triangle is given by one-half the product of the base by the altitude," would cease to be true if, instead of taking as the unit of measure of areas the square with side of unit length, one were to take the triangle having such unit as base and altitude. These considerations are intimately connected with those by which the pragmatists have been led to a more precise determination of the difference expressed in ordinary speech by opposing "laws" to facts; and to put on an entirely new basis the classic controversy between determinists and freedomists. (See *Leonardo*, April, 1905, p. 57, and Poincaré, *Valeur de la science*.)

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A third point of contact between pragmatists and mathematical logicians consists in the interest shown on both sides for historical researches in the development of scientific theories. Both consider these essential as a means of recognizing the equivalence or identity of theories, under the various forms which they have assumed in different epochs or fields—all the time expressing substantially the same facts and serving the same ends. The logicians as well as the pragmatists have thus contributed to destroy



a number of prejudices attributed to supposed incompatibilities between the theories now current and the views of the great scientists or thinkers of antiquity. The fact has been brought to light that many, and those not the least important, among the discoveries of modern mathematicians, have consisted in nothing more than the introduction of new methods or notation for processes formerly employed or considered under other names by their predecessors. In the *Formulario* of Peano the importance given to historical data has steadily increased, especially under the inspiration of one of the principal collaborators, Vacca (among other things an enthusiastic investigator of the development of mathematics in the Far East); and the importance attributed to articles of this kind now constitutes one of the most noteworthy among the distinctive characteristics of the method of treatment of the various branches of mathematics that the said *Formulario* presents. Theories are therein expounded, not as in the ordinary treatment, under their "static" aspect—as one might express it,—their aspect of repose; but under that of movement and development—not in the conventional attitudes of stuffed animals, with glass eyes; but as organisms, which live, eat, struggle, reproduce: or at least like figures in a cinematograph, with some naturalness of progression and development.

To this tendency to recognize the identity of theories, beyond or under differences of expression, symbolism, language, representative conventions and the rest, is to be attributed also the constant interest of the mathematical logicians in linguistic questions—from Grassmann, at once the author of the *Ausdehnungslehre* and of the *Wörterbuch zum Rig-Veda*, to Nagy, student of the transmission of Greek thought through the Syriac and Arabic commentaries; from Couturat, joint author with Leau of a *History of the Projects of "Universal Language,"* to

Peano, inventor and propagandist of one of the most practical among them: the "*latino non flexo*."

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Quite a different series of relations between pragmatists and mathematical logicians is offered by the important progress made by the latter in the theory of "definition."

First of all, the traditional method which makes definitions consist in the search for *genera* and *specific differences*, i. e., in the search for classes from which the class to be defined may result through the mediation of a "logical product," has been broadened so as to include every case in which the class to be defined may be obtained as a *function* of known classes, by means of any previously admitted operation or series of operations.

In another direction the scholastic methods of definition have been broadened by taking into consideration the cases in which that which is defined is not an isolated word but a group of words or phrases in which this word appears (*implicit definitions*). Hereby we have come to recognize more clearly than did, say, Aristotle, that definitions of isolated words are only a particular case, the simplest, in the vaster field of "implicit definitions." We see, for example, that to define a noun *A* signifies nothing more nor less than to indicate the sense which would be attributed to the phrase: "this thing or that is an *A*." Moreover it has become possible to characterize and justify the procedure, already instinctively followed by mathematicians, of employing successively diverse definitions of one and the same notation, according to the fields (whether inclusive or not) in which arises the opportunity of making use of groups of symbols in which this notation figures (definitions preceded by hypotheses limiting, and varying with, the variation of the definitions).

Particular interest, in their relations to pragmatism, is presented by what were called (Peano) "*definitions by*

*abstraction*"; in which, from the fact that a given relation presents some of the characteristic properties of equality, occasion is taken to fashion a new concept: as, for example, from the fact that two straight lines parallel to a third are parallel to each other, is drawn the concept "direction"; or, from the fact that two amounts of merchandise exchanged for one and the same amount of a third commodity are mutually exchangeable, is evolved the concept "value."

A character common to the latter and to the other above-mentioned innovation, among those introduced by the mathematical logicians into the traditional theory of definition, consists in their tendency to bring to light the various orders of circumstances upon which may depend the fact that of a given word, taken by itself, a definition in the ordinary sense cannot be given; that is, a phrase cannot be enunciated which will indicate directly the characteristic or characteristics belonging to the objects to which the word in question is applied.

Not only has mathematical logic led to a recognition of the fact that to speak of the "definability" or "non-definability" of a given word or concept is to use a meaningless phrase, so long as no precise indication is given as to what *other words* or concepts may be used in the desired definition; but it has also afforded an explanation of the fact that many among the most important words of science and philosophy are found to be among those very ones of which it is unreasonable to ask or to seek a definition, in the scholastic sense. Mathematical logic has thus contributed most efficaciously to the defense of the position of the pragmatists against the "agnostic" prejudice which attributes the impossibility of the resolution of such problems to a pretended incapacity of the human mind to penetrate the "essence" of things.

The so-called "definitions by postulates"—i. e., those

which consist in determining the significance of a sign of operation or of relation by enunciating a certain number of norms which, by hypothesis, are to regulate its application—have, on the other hand, affinities with pragmatism in that they conduce to a clearer recognition of the arbitrary character of postulates, as well as of definitions. These appear in their true quality as propositions which possess the function of determining, in view of given ends or applications, the various fields of research; that is, as propositions whose sole justification consists in the importance and utility of the *consequences* which it may be possible to deduce therefrom.

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Another characteristic of mathematical logic, in which, perhaps even more than in any of the afore-mentioned, is shown its affinity to pragmatism, is that which relates to the function which has come to be assumed by the search for and construction of "particular interpretations," or concrete examples, as criteria for determining the mutual independence, or the compatibility, of given affirmations or hypotheses.

Originally considered as simply a means of ascertaining the *necessity* (indispensability) of given premises, or the impossibility of reaching determinate conclusions without their aid, this search for particular examples has come to be regarded as the *only* process capable of guaranteeing that any group of hypotheses does not contain "implicit contradictions." That is, the construction of concrete interpretations, by which all the premises or hypotheses underlying a given deductive theory may be simultaneously verified, has assumed the importance of a condition in the absence of which even the most rigorous lines of reasoning can lead only to conclusions liable to contradiction by others, which may be obtained through deductions not less rigorous *than the premises themselves*.

Furthermore, in the choice of examples *hierarchies* have come to be formed, according to their degree of concreteness and determinateness. To such of these as are the most concrete and determinate of all—i. e., to the examples which belong to the field of arithmetic—has been attributed for the above-mentioned purpose, by some, a superiority over all others; particularly over those which imply considerations of continuity, or which belong to fields in which it is found more difficult to effect an exact and complete characterization or formulation of the facts adduced.

In this need of reinforcement by particular facts which is *inherent* in the more abstract theories (and this need is in direct proportion to their abstractness)—not, indeed, a need of facts which shall serve to confirm or to render inductively probable the separate premises upon which they are based; but of facts which shall guarantee the possibility of the *coexistence* and *cooperation* of such premises:—in this need of pure logic to derive strength, like Antæus, from periodic contact with the earth, one cannot fail to recognize one of the most significant indications of that mysterious alliance between “the extremes of theoretic activity” (the intuition of the *particular* and the impulse to abstraction and generalization) which it is not the least of the merits of the pragmatic theories to have noted and proclaimed.\*

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Pragmatists and mathematicians find themselves in agreement, too, in their efforts toward the maximum of *conciseness* and *rapidity* of expression—in their tendency to eliminate all superfluity and redundancy both of wording and of concept.

Both find the value of theories and doctrines not only *in that which is said* but also in what is *unmentioned* and

\* Cf. G. Papini, “Les extrêmes de l’activité théorique” (in *Comptes rendus du IIe congrès internationale de philosophie.*) Geneva: Kündig, 1905.

whereof exposition or consideration is suppressed. See the article of Giuliano il Sofista on the Nourishment of Fasting, (*Leonardo*, April, 1905).

One of the principal achievements of mathematical logic consists in this very recognition of the fact that so many of what pass for *mathematical truths* owe their existence solely to imperfections of notation, which permit the enunciation of the same fact in different ways—to have afterward the pleasure of recognizing it as one and the same under its divers expressions. An example of this is to be found in the propositions of trigonometry, which re-enunciate, in new garb, theorems of elementary geometry; and, furthermore, re-enunciate them in manifold forms, of which the trigonometrical identities do no more than to express the equivalence.

By the introduction of other new symbols the number of "truths" of this kind might be increased indefinitely—repeating in science the miracle of the multiplication of the loaves and fishes; with the difference only that the results thus obtained would serve much more to distend than to nourish the minds to which they should be communicated. Indeed in this connection, as my friend G. Vacca observes, one might enunciate a law in form analogous to Malthus's Law, consisting in this: that when the concepts or the words which are introduced into a theory increase in arithmetical ratio, the corresponding propositions—whose truth or falsity the "science," to be complete, must needs decide—increase more rapidly than any geometrical progression (following an exponential law, enunciated by Clifford. See Peano, *Calcolo geometrico*, 1888).

Against a similar fatty degeneration of theories pragmatism, likewise, represents an energetic reaction; insisting as it does on the *instrumental* character of theories—affirming that they are not an *end in themselves*, but *media* and "organisms" whose efficacy and value is rigorously

dependent upon their agility, upon the absence of encumbrances and hindrances to their movements, upon their resemblance rather to lions and tigers than to hippopotami and mastodons. The favorite dictum of Plato: *ρεῖτρον ἡμῶν παντός* is no less applicable to scientific theories than to any other branch of human activity.

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## PROLEGOMENA TO AN APOLOGY FOR PRAG- MATICISM.

COME on, my Reader, and let us construct a diagram to illustrate the general course of thought; I mean a System of diagrammatization by means of which any course of thought can be represented with exactitude.

"But why do that, when the thought itself is present to us?" Such, substantially, has been the interrogative objection raised by more than one or two superior intelligences, among whom I single out an eminent and glorious General.

Recluse that I am, I was not ready with the counter-question, which should have run, "General, you make use of maps during a campaign, I believe. But why should you do so, when the country they represent is right there?" Thereupon, had he replied that he found details in the maps that were so far from being "right there," that they were within the enemy's lines, I ought to have pressed the question, "Am I right, then, in understanding that, if you were thoroughly and perfectly familiar with the country, as, for example, if it lay just about the scenes of your childhood, no map of it would then be of the smallest use to you in laying out your detailed plans?" To that he could only have rejoined, "No, I do not say that, since I might probably desire the maps to stick pins into, so as to mark each anticipated day's change in the situations of the two armies." To that again, my sur-

rejoinder should have been, "Well, General, that precisely corresponds to the advantages of a diagram of the course of a discussion. Indeed, just there, where you have so clearly pointed it out, lies the advantage of diagrams in general. Namely, if I may try to state the matter after you, one can make exact experiments upon uniform diagrams; and when one does so, one must keep a bright lookout for unintended and unexpected changes thereby brought about in the relations of different significant parts of the diagram to one another. Such operations upon diagrams, whether external or imaginary, take the place of the experiments upon real things that one performs in chemical and physical research. Chemists have ere now, I need not say, described experimentation as the putting of questions to Nature. Just so, experiments upon diagrams are questions put to the Nature of the relations concerned." The General would here, may be, have suggested, (if I may emulate illustrious warriors in reviewing my encounters in afterthought,) that there is a good deal of difference between experiments like the chemist's, which are trials made upon the very substance whose behavior is in question, and experiments made upon diagrams, these latter having no physical connection with the things they represent. The proper response to that, and the only proper one, making a point that a novice in logic would be apt to miss, would be this: "You are entirely right in saying that the chemist experiments upon the very object of investigation, albeit, after the experiment is made, the particular sample he operated upon could very well be thrown away, as having no further interest. For it was not the particular sample that the chemist was investigating; it was the molecular *structure*. Now he was long ago in possession of overwhelming proof that all samples of the same molecular structure react chemically in exactly the same way; so that one sample

is all one with another. But the object of the chemist's research, that upon which he experiments, and to which the question he puts to Nature relates, is the Molecular Structure, which in all his samples has as complete an identity as it is in the nature of Molecular Structure ever to possess. Accordingly, he does, as you say, experiment upon the Very Object under investigation. But if you stop a moment to consider it, you will acknowledge, I think, that you slipped in implying that it is otherwise with experiments made upon diagrams. For what is there the Object of Investigation? It is the *form of a relation*. Now this Form of Relation is the very form of the relation between the two corresponding parts of the diagram. For example, let  $f_1$  and  $f_2$  be the two distances of the two foci of a lens from the lens. Then,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_o}.$$

This equation is a diagram of the form of the relation between the two focal distances and the principal focal distance; and the conventions of algebra (and all diagrams, nay all pictures, depend upon conventions) in conjunction with the writing of the equation, establish a relation between the very *letters*  $f_1, f_2, f_o$ , regardless of their significance, the form of which relation is the *Very Same* as the form of the relation between the three focal distances that these letters denote. This is a truth quite beyond dispute. Thus, this algebraic Diagram presents to our observation the very, identical object of mathematical research, that is, the Form of the harmonic mean, which the equation aids one to study. [But do not let me be understood as saying that a Form possesses, itself, Identity in the strict sense; that is, what the logicians, translating  $\alpha\rho\iota\theta\mu\epsilon\acute{\omega}$ , call "numerical identity."]

Not only is it true that by experimentation upon some diagram an experimental proof can be obtained of every

necessary conclusion from any given Copulate of Premises, but, what is more, no "necessary" conclusion is any more apodictic than inductive reasoning becomes from the moment when experimentation can be multiplied *ad libitum* at no more cost than a summons before the imagination. I might furnish a regular proof of this, and am dissuaded from doing so now and here only by the exigency of space, the ineluctable length of the requisite explanations, and particularly by the present disposition of logicians to accept as sufficient F. A. Lange's persuasive and brilliant, albeit defective and in parts even erroneous, apology for it. Under these circumstances, I will content myself with a rapid sketch of my proof. First, an analysis of the essence of a sign, (stretching that word to its widest limits, as *anything which, being determined by an object, determines an interpretation to determination, through it, by the same object,*) leads to a proof that every sign is determined by its object, either first, by partaking in the characters of the object, when I call the sign an *Icon*; secondly, by being really and in its individual existence connected with the individual object, when I call the sign an *Index*; thirdly, by more or less approximate certainty that it will be interpreted as denoting the object, in consequence of a habit [which term I use as including a natural disposition], when I call the sign a *Symbol*.\* I next examine into the different efficiencies and inefficiencies of these three kinds of signs in aiding the ascertainment of truth. A Symbol incorporates a habit, and is indispensable to the application of any *intellectual* habit, *at least*. Moreover, Symbols afford the means of thinking about thoughts in ways in which we could not otherwise think of them. They enable us, for example, to create Abstractions, without which we should lack

\* In the original publication of this division, in 1867, the term "representamen" was employed in the sense of a sign in general, while "sign" was taken as a synonym of *index*, and an *Icon* was termed a "likeness."

a great engine of discovery. These enable us to count, they teach us that collections are individuals [individual = individual object], and in many respects they are the very warp of reason. But since symbols rest exclusively on habits already definitely formed but not furnishing any observation even of themselves, and since knowledge is habit, they do not enable us to add to our knowledge even so much as a necessary consequent, unless by means of a definite preformed habit. Indices, on the other hand, furnish positive assurance of the reality and the nearness of their Objects. But with the assurance there goes no insight into the nature of those Objects. The same Perceptible may, however, function doubly as a Sign. That footprint that Robinson Crusoe found in the sand, and which has been stamped in the granite of fame, was an Index to him that some creature was on his island, and at the same time, as a Symbol, called up the idea of a man. Each *Icon* partakes of some more or less overt character of its object. They, one and all, partake of the most overt character of all lies and deceptions, — their Overtness. Yet they have more to do with the living character of truth than have either Symbols or Indices. The Icon does not stand unequivocally for this or that existing thing, as the Index does. Its Object may be a pure fiction, as to its existence. Much less is its Object necessarily a thing of a sort habitually met with. But there is one assurance that the Icon does afford in the **highest degree**. Namely, that which is displayed before the mind's gaze, — the Form of the Icon, which is also its object, — must be *logically possible*. This division of Signs is only one of ten different divisions of Signs which I have found it necessary more especially to study. I do not say that they are all satisfactorily definite in my mind. They seem to be all trichotomies, which form an attribute to the essentially triadic nature of a Sign. I mean because

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three things are concerned in the functioning of a Sign; the Sign itself, its Object, and its Interpretant. I cannot discuss all these divisions in this article; and it can well be believed that the whole nature of reasoning cannot be fully exposed from the consideration of one point of view among ten. That which we can learn from this division is of what sort a Sign must be to represent the sort of Object that reasoning is concerned with. Now reasoning has to make its conclusion manifest. Therefore, it must be chiefly concerned with forms, which are the chief objects of rational insight. Accordingly, Icons are specially requisite for reasoning. A Diagram is mainly an Icon, and an Icon of intelligible relations. It is true that what must be is not to be learned by simple inspection of anything. But when we talk of deductive reasoning being necessary, we do not mean, of course, that it is infallible. But precisely what we do mean is that the conclusion follows from the form of the relations set forth in the premiss. Now since a diagram, though it will ordinarily have Symbolide Features, as well as features approaching the nature of Indices, is nevertheless in the main an Icon of the forms of relations in the constitution of its Object, the appropriateness of it for the representation of necessary inference is easily seen. But since you may, perhaps, be puzzled to understand how an Icon can exhibit a necessity—a Must-be,—I will here give, as an example of its doing so, my proof that the single members of no collection or plural, are as many as are the collections it includes, each reckoned as a single object, or, in other words, that there can be no relation in which every collection composed of members of a given collection should (taken collectively as a single object,) stand to some member of the latter collection to which no other such included collection of the following proposition, namely: that, taking any collection or plural, whatsoever, be it finite or infinite, and

calling this the *given collection*; and considering all the collections, or plurals, each of which is composed of some of the individual members of the given collection (but including along with these *Nothing* which is to be here as a collection having no members at all; and also including the single members of the given collection, conceived as so many collections each of a single member), and calling these the *involved collections*; the proposition is that there is no possible relation in which each involved collection, (considered as a single object,) stands to a member of the given collection, without any other of the involved collections standing in the same relation to that same member of the given collection, so stands. This purely symbolic statement can be rendered much more perspicuous by the introduction of Indices, as follows. The proposition is that no matter what collection  $C$  may be, and no matter what relation  $R$  may be, there must be some collection,  $c'$ , composed exclusively of members of  $C$ , which does not stand in the relation  $R$  to any member,  $k$ , of  $C$ , unless some other collection,  $c''$ , likewise composed of members of  $C$ , stands in the same relation  $R$  to the same  $k$ . The theorem is important in the doctrine of multitude, since it is the same as to say that any collection, no matter how great, is less multitudinous than the collection of possible collections composed exclusively of members of it; although formerly this was assumed to be false of some infinite collections. The demonstration begins by insisting that, if the proposition be false, there must be some definite relation of which it is false. Assume, then, that the letter  $R$  is an index of any one such relation you please. Next divide the members of  $C$  into four classes as follows:

Class I is to consist of all those members of  $C$  (if there be any such) to each of which no collection of members of  $C$  stands in the relation  $R$ .

Class II is to consist of all those members of  $C$  to



each of which one and only one collection of members of  $C$  stands in the relation  $R$ ; and this class has two subclasses, as follows:

Sub-Class 1 is to consist of whatever members of Class II there may be each of which is contained in that one collection of members of  $C$  that is in the relation,  $R$ , to it.

Sub-Class 2 is to consist of whatever members of Class II there may be none of which is contained in that one collection of members of  $C$  that is in the relation  $R$  to it.

Class III is to consist of all those members of  $C$ , if there be any such, to each of which more than one collection of members of  $C$  are in the relation  $R$ .

This division is complete; but everybody would consider the easy diagrammatical proof that it is so as needless to the point of nonsense, implicitly relying on a Symbol in his memory which assures him that every Division of such construction is complete.

I ought already to have mentioned that, throughout the enunciation and demonstration of the proposition to be proved, the term "collection included in the given collection" is to be taken in a peculiar sense to be presently defined. It follows that there is one "possible collection" that is included in every other, that is, which excludes whatever any other excludes. Namely, this is the "possible collection" which includes only the Sphinxes, which is the same that includes only the Basilisks, and is identical with the "possible collection" of all the Centaurs, the unique and ubiquitous collection called "Nothing," which has no member at all. If you object to this use of the term "collection," you will please substitute for it, throughout the enunciation and the demonstration, any other designation of the same object. I prefix the adjective "pos-

sible," though I must confess it does not express my meaning, merely to indicate that I extend the term "collection" to Nothing, which, of course, has no existence. Were the suggested objection to be persisted in by those *soi-disant* reasoners who refuse to think at all about the object of this or that description, on the ground that it is "inconceivable," I should not stop to ask them how they could say that, when that involves thinking of it in the very same breath, but should simply say that for them it would be necessary to except collections consisting of single individuals. Some of these mighty intellects refuse to allow the use of any name to denote single individuals and also plural collections along with them; and for them the proposition ceases to be true of pairs. If they would not allow pairs to be denoted by any term that included all higher collections, the proposition would cease to be true of triplets and so on. In short, by restricting the meaning of "possible collection," the proposition may be rendered false of *small* collections. No general formal restriction can render it false of *greater* collections.

I shall now assume that you will permit me to use the term "possible collection" according to the following definition. A "possible collection" is an *ens rationis* of such a nature that the definite plural of any noun, or possible noun of definite signification, (as "the A's," "the B's," etc) denotes one, and only one, "possible collection" in any one perfectly definite state of the universe; and there is a certain relation between some "possible collections," expressed by saying that one "possible collection" *includes* another (or the same) "possible collection," and if, and only if, of two nouns one is universally and affirmatively predicable of the other in any one perfectly definite state of the universe, then the "possible collection" denoted by the definite plural of the former *includes* whatever "possible collection" is *included* by the "possible collection"

denoted by the definite plural of the latter, and of any two different "possible collections," one or other must *include* something not *included* by the other.

A diagram of the definition of "possible collection" being compared with a diagram embracing whatever members of subclasses 1 and 2 that it may, excluding all the rest, will now assure us that any such aggregate is a possible collection of members of the class C, no matter what individuals of Classes I and III be included or excluded in the aggregate along with those members of Class II, if any there be in the aggregate.

We shall select, then, a single possible collection of members of C to which we give the proper name *c*, and this possible collection shall be one which contains no individual of Subclass 1, but contains whatever individual there may be of Subclass 2. We then ask whether or not it is true that *c* stands in the relation *R* to a member of C to which no other possible collection of members of C stands in the same relation; or, to put this question into a more convenient shape, we ask, Is there any member of the Class C to which *c* and no other possible collection of members of C stands in the relation *R*? If there be such a member or members of C, let us give one of them the proper name T. Then T must belong to one of our four divisions of this class. That is,

either T belongs to Class I, (but that cannot be since by the definition of Class I, to no member of this class is any possible collection of members of C in the relation *R*);

or T belongs to Subclass 1, (but that cannot be, since by the definition of that subclass, every member of it is a member of the only possible collection of members of C that is *R* to it, which possible collection cannot be *c*, because *c* is only known to us by a description which forbids its containing any

member of Subclass 1. Now it is  $c$ , and  $c$  only, that is in the relation  $R$  to  $T$ );

or  $T$  belongs to Subclass 2, (but that cannot be, since by the definition of that subclass, no member of it is a member of the only possible collection of members of  $C$  that is  $R$  to it, which possible collection cannot be  $c$ , because the description by which alone  $c$  can be recognized makes it contain every member of Subclass 2. Now it is  $c$  only that is in the relation  $R$  to  $T$ );

or  $T$  belongs to Class III (but this cannot be, since to every member of that class, by the definition of it, more than one collection of members of  $C$  stand in the relation  $R$ , while to  $T$  only one collection, namely,  $c$ , stands in that relation).

Thus,  $T$  belongs to none of the classes of members of  $C$ , and consequently is not a member of  $C$ . Consequently, there is no such member of  $C$ ; that is, no member of  $C$  to which  $c$ , and no other possible collection of members of  $C$ , stands in the relation  $R$ . But  $c$  is the proper name we were at liberty to give to whatever possible collection of members of  $C$  we pleased. Hence, there is no possible collection of members of  $C$  that stands in the relation  $R$  to a member of the class  $C$  to which no other possible collection of members of  $C$  stands in this relation  $R$ . But  $R$  is the name of *any* relation we please, and  $C$  is any class we please. It is, therefore, proved that no matter what class be chosen, or what relation be chosen, there will be some possible collection of members of that class ( in the sense in which Nothing is such a collection) which does not stand in that relation to any member of that class to which no other such possible collection stands in the same relation.

When I was a boy, my logical bent caused me to take pleasure in tracing out upon a map of an imaginary laby-

rynth one path after another in hopes of finding my way to a central compartment. The operation we have just gone through is essentially of the same sort, and if we are to recognize the one as essentially performed by experimentation upon a diagram, so must we recognize that the other is performed. The demonstration just traced out brings home to us very strongly, also, the convenience of so constructing our diagram as to afford a clear view of the mode of connection of its parts, and of its composition at each stage of our operations upon it. Such convenience is obtained in the diagrams of algebra. In logic, however, the desirability of convenience in threading our way through complications is much less than in mathematics, while there is another desideratum which the mathematician as such does not feel. The mathematician wants to reach the conclusion, and his interest in the process is merely as a means to reach similar conclusions. The logician does not care what the result may be; his desire is to understand the nature of the process by which it is reached. The mathematician seeks the speediest and most abridged of secure methods; the logician wishes to make each smallest step of the process stand out distinctly, so that its nature may be understood. He wants his diagram to be, above all, as analytical as possible.

In view of this, I beg leave, Reader, as an Introduction to my defence of pragmatism, to bring before you a very simple system of diagrammatization of propositions which I term the System of Existential Graphs. For, by means of this, I shall be able almost immediately to deduce some important truths of logic, little understood hitherto, and closely connected with the truth of pragmatism; while discussions of other points of logical doctrine, which concern pragmatism but are not directly settled by this system, are nevertheless much facilitated by reference to it.

By a *graph*, (a word overworked of late years,) I, for

my part, following my friends Clifford and Sylvester, the introducers of the term, understand in general a diagram composed principally of spots and of lines connecting certain of the spots. But I trust it will be pardoned to me that, when I am discussing Existential Graphs, without having the least business with other Graphs, I often omit the differentiating adjective and refer to an Existential Graph as a Graph simply. But you will ask, and I am plainly bound to say, precisely what kind of a Sign an Existential Graph, or as I abbreviate that phrase here, a *Graph*, is. In order to answer this I must make reference to two different ways of dividing all Signs. It is no slight task, when one sets out from none too clear a notion of what a Sign is,—and you will, I am sure, Reader, have noticed that my definition of a Sign is not convincingly distinct,—to establish a single vividly distinct division of all Signs. The one division which I have already given has cost more labor than I should care to confess. But I certainly could not tell you what sort of a Sign an Existential Graph is, without reference to two other divisions of Signs. It is true that one of these involves none but the most superficial considerations, while the other, though a hundredfold more difficult, resting as it must for a clear comprehension of it upon the profoundest secrets of the structure of Signs, yet happens to be extremely familiar to every student of logic. But I must remember, Reader, that your conceptions may penetrate far deeper than mine; and it is to be devoutly hoped they may. Consequently, I ought to give such hints as I conveniently can, of my notions of the structure of Signs, even if they are not strictly needed to express my notions of Existential Graphs.

I have already noted that a Sign has an Object and an Interpretant, the latter being that which the Sign produces in the Quasi-mind that is the Interpreter by determining the latter to a feeling, to an exertion, or to a Sign, which

determination is the Interpretant. But it remains to point out that there are usually two Objects, and more than two Interpretants. Namely, we have to distinguish the Immediate Object, which is the Object as the Sign itself represents it, and whose Being is thus dependent upon the Representation of it in the Sign, from the Dynamical Object, which is the Reality which by some means contrives to determine the Sign to its Representation. In regard to the Interpretant we have equally to distinguish, in the first place, the Immediate Interpretant, which is the interpretant as it is revealed in the right understanding of the Sign itself, and is ordinarily called the *meaning* of the sign; while in the second place, we have to take note of the Dynamical Interpretant which is the actual effect which the Sign, as a Sign, really determines. Finally there is what I provisionally term the Final Interpretant, which refers to the manner in which the Sign tends to represent itself to be related to its Object. I confess that my own conception of this third interpretant is not yet quite free from mist. Of the ten divisions of signs which have seemed to me to call for my special study, six turn on the characters of an Interpretant and three on the characters of the Object. Thus the division into Icons, Indices, and Symbols depends upon the different possible relations of a Sign to its Dynamical Object. Only one division is concerned with the nature of the Sign itself, and this I now proceed to state.

A common mode of estimating the amount of matter in a MS. or printed book is to count the number of words.\* There will ordinarily be about twenty *thes* on a page, and of course they count as twenty words. In another sense of the word "word," however, there is but one word "the" in the English language; and it is impossible that this word should lie visibly on a page or be heard in any voice.

\* Dr. Edward Eggleston originated the method.



for the reason that it is not a Single thing or Single event. It does not exist; it only determines things that do exist. Such a definitely significant Form, I propose to term a *Type*. A Single event which happens once and whose identity is limited to that one happening or a Single object or thing which is in some single place at any one instant of time, such event or thing being significant only as occurring just when and where it does, such as this or that word on a single line of a single page of a single copy of a book, I will venture to call a *Token*. An indefinite significant character such as a tone of voice can neither be called a Type nor a Token. I propose to call such a Sign a *Tone*. In order that a Type may be used, it has to be embodied in a Token which shall be a sign of the Type, and thereby of the object the Type signifies. I propose to call such a Token of a Type an *Instance* of the Type. Thus, there may be twenty Instances of the Type "the" on a page. The term (Existential) *Graph* will be taken in the sense of a Type; and the act of embodying it in a *Graph-Instance* will be termed *scribing* the Graph (not the Instance), whether the Instance be written, drawn, or incised. A mere blank place is a Graph-Instance, and the Blank *per se* is a Graph; but I shall ask you to assume that it has the peculiarity that it cannot be abolished from any Area on which it is scribed, as long as that Area exists.

A familiar logical triplet is Term, Proposition, Argument. In order to make this a division of all signs, the first two members have to be much widened. By a *Seme*. I shall mean anything which serves for any purpose as a substitute for an object of which it is, in some sense, a representative or Sign. The logical Term, which is a class-name, is a Seme. Thus, the term "The mortality of man" is a Seme. By a *PHEME* I mean a Sign which is equivalent to a grammatical sentence, whether it be Interrogative, Imperative, or Assertory. In any case, such a

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Sign is intended to have some sort of compulsive effect on the Interpreter of it. As the third member of the triplet, I sometimes use the word *Delome* (pronounce deeloam, from  $\delta\eta\lambda\omega\mu\alpha$ ), though *Argument* would answer well enough. It is a Sign which has the Form of tending to act upon the Interpreter through his own self-control, representing a process of change in thoughts or signs, as if to induce this change in the Interpreter.

A Graph is a PHEME, and in my use hitherto, at least, a Proposition. An Argument is represented by a series of Graphs.

The Immediate Object of all knowledge and all thought is, in the last analysis, the Percept. This doctrine in no wise conflicts with Pragmaticism, which holds that the Immediate Interpretant of all thought proper is Conduct. Nothing is more indispensable to a sound epistemology than a crystal-clear discrimination between the Object and the Interpretant of knowledge; very much as nothing is more indispensable to sound notions of geography than a crystal-clear discrimination between north latitude and south latitude; and the one discrimination is not more rudimentary than the other. That we are conscious of our Percepts is a theory that seems to me to be beyond dispute; but it is not a fact of Immediate Perception. A fact of Immediate Perception is not a Percept, nor any part of a Percept; a Percept is a Seme, while a fact of Immediate Perception or rather the Perceptual Judgment of which such fact is the Immediate Interpretant, is a PHEME that is the direct Dynamical Interpretant of the Percept, and of which the Percept is the Dynamical Object, and is with some considerable difficulty, (as the history of psychology shows,) distinguished from the Immediate Object, though the distinction is highly significant. But not to interrupt our train of thought, let us go on to note that while the Immediate Object of a Percept is excessively vague, yet

natural thought makes up for that lack, (as it almost amounts to,) as follows. A late Dynamical Interpretant of the whole complex of Percepts is the Seme of a Perceptual Universe that is represented in instinctive thought as determining the original Immediate Object of every Percept. Of course, I must be understood as talking not psychology, but the logic of mental operations. Subsequent Interpretants furnish new Semes of Universes resulting from various adjunctions to the Perceptual Universe. They are, however, all of them, Interpretants of Percepts.

Finally, and in particular, we get a Seme of that highest of all Universes which is regarded as the Object of every true Proposition, and which, if we name it all, we call by the somewhat misleading title of "The Truth."

That said, let us go back and ask this question: How is it that the Percept, which is a Seme, has for its direct Dynamical Interpretant the Perceptual Judgment, which is a PHEME? For that is not the usual way with Semes, certainly. All the examples that happen to occur to me at this moment of such action of Semes are instances of Percepts, though doubtless there are others. Since not all Percepts act with equal energy in this way, the instances may be none the less instructive for being Percepts. However, Reader, I beg you will think this matter out for yourself, and then you can see,—I wish I could,—whether your independently formed opinion does not fall in with mine. My opinion is that a pure perceptual Icon,—and many really *great* psychologists have evidently thought that Perception is a passing of images before the mind's eye, much as if one were walking through a picture-gallery,—could not have a PHEME for its direct Dynamical Interpretant. I desire, for more than one reason, to tell you *why* I think so, although that you should to-day appreciate my reasons seems to be out of the question. Still.

I wish you to understand me so far as to know that, mistaken though I be, I am not so sunk in intellectual night as to be dealing lightly with philosophic Truth when I aver that weighty reasons have moved me to the adoption of my opinion; and I am also anxious that it should be understood that those reasons have not been psychological at all, but are purely logical. My reason, then, briefly stated and abridged, is that it would be *illogical* for a pure Icon to have a PHEME for its Interpretant, and I hold it to be impossible for thought not subject to self-control, as a Perceptual Judgment manifestly is not, to be illogical. I dare say this reason may excite your derision or disgust, or both; and if it does, I think none the worse of your intelligence. You probably opine, in the first place, that there is no meaning in saying that thought which draws no Conclusion is illogical, and that, at any rate, there is no standard by which I can judge whether such thought is logical or not; and in the second place, you probably think that, if self-control has any essential and important relation to logic, which I guess you either deny or strongly doubt, it can only be that it is that which makes thought *logical*, or else which establishes the distinction between the logical and the illogical, and that in any event it has to be such as it is, and would be logical, or illogical, or both, or neither, whatever course it should take. But though an Interpretant is not necessarily a Conclusion, yet a Conclusion is necessarily an Interpretant. So that if an Interpretant is not subject to the rules of Conclusions there is nothing monstrous in my thinking it is subject to some generalization of such rules. For any evolution of thought, whether it leads to a Conclusion or not, there is a certain normal course, which is to be determined by considerations not in the least psychological, and which I wish to expound in my next article; and while I entirely agree, in opposition to distinguished logicians, that normality can be no

criterion for what I call rationalistic reasoning, such as alone is admissible in science, yet it is precisely the criterion of instinctive or common-sense reasoning, which, within its own field, is much more trustworthy than rationalistic reasoning. In my opinion, it is self-control which makes any other than the normal course of thought possible, just as nothing else makes any other than the normal course of action possible; and just as it is precisely that that gives room for an ought-to-be of conduct, I mean Morality, so it equally gives room for an ought-to-be of thought, which is Right Reason; and where there is no self-control, nothing but the normal is possible. If your reflections have led you to a different conclusion from mine, I can still hope that when you come to read my next article, in which I shall endeavor to show what the forms of thought are, in general and in some detail, you may yet find that I have not missed the truth.

But supposing that I am right, as I probably shall be in the opinions of *some* readers, how then is the Perceptual Judgment to be explained? In reply, I note that a Percept cannot be dismissed at will, even from memory. Much less can a person prevent himself from perceiving that which, as we say, stares him in the face. Moreover, the evidence is overwhelming that the perceiver is aware of this compulsion upon him; and if I cannot say for certain how this knowledge comes to him, it is not that I cannot conceive how it could come to him, but that, there being several ways in which this might happen, it is difficult to say which of those ways actually is followed. But that discussion belongs to psychology; and I will not enter upon it. Suffice it to say that the perceiver is aware of being compelled to perceive what he perceives. Now existence means precisely the exercise of compulsion. Consequently, whatever feature of the percept is brought into relief by some association and thus attains a logical position like

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that of the observational premiss of an explaining Abduction,\* the attribution of Existence to it in the Perceptual Judgment is virtually and in an extended sense, a logical Abductive Inference nearly approximating to necessary inference. But my next paper will throw a flood of light upon the logical affiliation of the Proposition and the Pheme generally, to coercion.

That conception of Aristotle which is embodied for us in the cognate origin of the terms *actuality* and *activity* is one of the most deeply illuminating products of Greek thinking. Activity implies a generalization of *effort*; and effort is a two-sided idea, effort and resistance being inseparable, and therefore the idea of Actuality has also a dyadic form.

No cognition and no Sign is absolutely precise, not even a Percept; and indefiniteness is of two kinds, indefiniteness as to what is the Object of the Sign, and indefiniteness as to its Interpretant, or indefiniteness in Breadth and in Depth. Indefiniteness in Breadth may be either Implicit or Explicit. What this means is best conveyed in an example. The word *donation* is indefinite as to who makes the gift, what he gives, and to whom he gives it. But it calls no attention, itself, to this indefiniteness. The word *gives* refers to the same sort of fact, but its meaning is such that that meaning is felt to be incomplete unless those items are, at least formally, specified; as they are in "Somebody gives something to some person (real or artificial)." An ordinary Proposition ingeniously contrives to convey novel information through Signs whose significance depends entirely on the interpreter's familiarity with them; and this it does by means of a "Predicate," i. e., a term explicitly indefinite in breadth,

\* Abduction, in the sense I give the word, is any reasoning of a large class of which the provisional adoption of an explanatory hypothesis is the type. But it includes processes of thought which lead only to the suggestion of questions to be considered, and includes much besides.

and defining its breadth by means of "Subjects," or terms whose breadths are somewhat definite, but whose informative depth (i. e., all the depth except an essential superficiality) is indefinite, while conversely the depth of the Subjects is in a measure defined by the Predicate. A Predicate is either non-relative, or a *monad*, that is, is explicitly indefinite in one extensive respect, as is "black"; or it is a dyadic relative, or dyad, such as "kills," or it is a polyadic relative, such as "gives." These things must be diagrammatized in our system.

Something more needs to be added under the same head. You will observe that under the term "Subject" I include, not only the subject nominative, but also what the grammarians call the direct and the indirect object, together, in some cases, with nouns governed by prepositions. Yet there is a sense in which we can continue to say that a Proposition has but one Subject, for example, in the proposition, "Napoleon ceded Louisiana to the United States," we may regard as the Subject the ordered triplet, "Napoleon,—Louisiana,—the United States," and as the Predicate, "has for its first member, the agent, or party of the first part, for its second member the object, and for its third member the party of the second part of one and the same act of cession." The view that there are three subjects is, however, preferable for most purposes, in view of its being so much more analytical, as will soon appear.

All general, or definable, Words, whether in the sense of Types or of Tokens, are certainly Symbols. That is to say, they denote the objects that they do by virtue only of there being a habit that associates their signification with them. As to Proper Names, there might perhaps be a difference of opinion, especially if the Tokens are meant. But they should probably be regarded as Indices, since the actual connection (as we listen to talk,) of Instances



of the same typical words with the same Objects, alone causes them to be interpreted as denoting those Objects. Excepting, if necessary, propositions in which all the subjects are such signs as these, no proposition can be expressed without the use of Indices.\* If, for example, a man remarks, "Why, it is raining!" it is only by some such *circumstances* as that he is now standing here looking out at a window as he speaks, which would serve as an Index (not, however, as a Symbol,) that he is speaking of this place at this time, whereby we can be assured that he cannot be speaking of the weather on the satellite of Proeyon, fifty centuries ago. Nor are Symbols and Indices together generally enough. The arrangement of the words in the sentence, for instance, must serve as *Icons*, in order that the sentence may be understood. The chief need for the Icons is in order to show the Forms of the synthesis of the elements of thought. For in precision of speech, Icons can represent nothing but Forms and Feelings. That is why Diagrams are indispensable in all Mathematics, from Vulgar Arithmetic up, and in Logic are almost so. For Reasoning, nay, Logic generally, hinges entirely on Forms. You, Reader, will not need to be told that a regularly stated Syllogism is a Diagram; and if you take at random a half dozen out of the hundred odd logicians who plume themselves upon not belonging to the sect of Formal Logic, and if from this latter sect you take another half dozen at random, you will find that in proportion as the former avoid diagrams, they utilize the syntactical Form of their sentences. No pure Icons represent anything but Forms; no pure Forms are represented by anything but Icons. As for Indices, their utility especially shines where other Signs fail. Extreme precision being desired in the description of a red color, should I call it vermillion, I may be criti-

\* Strictly pure Symbols can signify only things familiar, and those only in so far as they are familiar.

cized on the ground that vermilion differently prepared has quite different hues, and thus I may be driven to the use of the color-wheel, when I shall have to Indicate four disks individually, or I may say in what proportions light of a given wave-length is to be mixed with white light to produce the color I mean. The wave-length being stated in fractions of a micron, or millionth of a meter, is referred through an Index to two lines on an individual bar in the Pavillon de Breteuil, at a given temperature and under a pressure measured against gravity at a certain station and (strictly) at a given date, while the mixture with white, after white has been fixed by an Index of an individual light, will require at least one new Index. But of superior importance in Logic is the use of Indices to denote Categories and Universes,\* which are classes that, being enormously large, very promiscuous, and known but in small part, cannot be satisfactorily defined, and therefore can only be denoted by Indices. Such, to give but a single instance, is the collection of all things in the Physical Universe. If anybody, your little son for example, who is such an assiduous researcher, always asking, What is the Truth, (*Τί ἐστὶν ἀλήθεια*;) but like "jesting Pilate," will not always stay for an answer, should ask you what the Universe of things physical is, you may, if convenient, take him to the Rigi-Kulm, and about sunset, point out all that is to be seen of Mountains, Forests, Lakes, Castles, Towns, and then, as the stars come out, all there is to be seen in the heavens, and all that though not seen, is reasonably conjectured to be there; and then tell him, "Imagine that what is to be seen in a city back yard to grow to all you can see here, and then let this grow in the same proportion as many times as there are trees in sight from

\* I use the term *Universe* in a sense which excludes many of the so-called "universes of discourse" of which Boole, De Morgan, and many subsequent logicians speak, but which, being perfectly definable, would in the present system be denoted by the aid of a graph.

here, and what you would finally have would be harder to find in the Universe than the finest needle in America's yearly crop of hay." But such methods are perfectly futile: Universes cannot be described.

Oh, I overhear what you are saying, O Reader: that a Universe and a Category are not at all the same thing; a Universe being a receptacle or class of Subjects, and a Category being a mode of Predication, or class of Predicates. I never said they were the same thing; but whether you describe the two correctly is a question for careful study.

Let us begin with the question of Univeres. It is rather a question of an advisable point of view than of the truth of a doctrine. A logical universe is, no doubt, a collection of *logical* subjects, but not necessarily of meta-physical Subjects, or "substances"; for it may be composed of characters, of elementary facts, etc. See my definition in Baldwin's Dictionary. Let us first try whether we may not assume that there is but one kind of Subjects which are either existing things or else quite fictitious. Let it be asserted that there is some married woman who will commit suicide in case her husband fails in business. Surely that is a very different proposition from the assertion that some married woman will commit suicide if all married men fail in business. Yet if nothing is real but existing things, then, since in the former proposition nothing whatever is said as to what the lady will or will not do if her husband does *not* fail in business, and since of a given married couple this can only be false if the fact is contrary to the assertion, it follows it can only be false if the husband *does* fail in business and if the wife then fails to commit suicide. But the proposition only says that there is *some* married couple of which the wife is of that temper. Consequently, there are only two ways in which the proposition can be false, namely, first, by there not

being any married couple, and secondly, by *every* married man failing in business while *no* married woman commits suicide. Consequently, all that is required to make the proposition true is that there should either be some married man who does not fail in business, or else some married woman who commits suicide. That is, the proposition amounts merely to asserting that there is a married woman who will commit suicide if *every* married man fails in business. The equivalence of these two propositions is the absurd result of admitting no reality but existence. If, however, we suppose that to say that a woman will suicide if her husband fails, means that every *possible* course of events would either be one in which the husband would not fail or one in which the wife would commit suicide, then, to make that false it will not be requisite for the husband actually to fail, but it will suffice that there are *possible* circumstances under which he would fail, while yet his wife would not commit suicide. Now you will observe that there is a great difference between the two following propositions:

- 1st, There is some *one* married woman who under all possible conditions would commit suicide or else her husband would not have failed.
- 2nd, Under all possible circumstances there is some married woman *or other* who would commit suicide, or else her husband would not have failed.

The former of these is what is really meant by saying that there is some married woman who would commit suicide if her husband were to fail, while the latter is what the denial of any possible circumstances except those that really take place logically leads to interpreting, (or virtually interpreting,) the Proposition as asserting.

In other places, I have given many other reasons for my firm belief that there are real possibilities. I also think, however, that, in addition to actuality and possi-

bility, a *third* mode of reality must be recognized in that which, as the gipsy fortune-tellers express it, is "sure to come true," or, as we may say, is *destined*,\* although I do not mean to assert that this is affirmation rather than the negation of this Mode of Reality. I do not see by what confusion of thought anybody can persuade himself that he does not believe that to-morrow is destined to come. The point is that it is to-day really true that to-morrow the sun will rise; or that, even if it does not, the clocks or *something*, will go on. For if it be not real it can only be fiction: a Proposition is either True or False. But we are too apt to confound destiny with the impossibility of the opposite. I see no impossibility in the sudden stoppage of everything. In order to show the difference, I remind you that, "impossibility" is that which, for example, describes the mode of falsity of the idea that there should be a collection of objects so multitudinous that there would not be characters enough in the universe of characters to distinguish all those things from one another. Is there anything of that sort about the stoppage of all motion? There is, perhaps, a *law of nature* against it; but that is all. However, I will postpone the consideration of that point. Let us, at least, *provide* for such a mode of being in our system of diagrammatization, since it *may* turn out to be needed and, as I think, surely will.

I will proceed to explain why, although I am not prepared to deny that every proposition can be represented, and that I must say, for the most part very conveniently, under your view that the Universes are receptacles of the Subjects alone, I, nevertheless, cannot deem that mode of analyzing propositions to be satisfactory.

\*I take it that anything may fairly be said to be *destined* which is sure to come about although there is no necessitating reason for it. Thus, a pair of dice, thrown often enough, will be sure to turn up sixes some time, although there is no necessity that they should. The probability that they will is 1: that is all. *Fate* is that special kind of *destiny* by which events are supposed to be brought about *under definite circumstances* which involve no necessitating cause for those occurrences.

And to begin with, I trust you will all agree with me that no analysis, whether in logic, in chemistry, or in any other science, is satisfactory, unless it be thorough, that is, unless it separates the compound into components each entirely homogeneous in itself, and therefore free from the smallest admixture of any of the others. It follows that in the Proposition, "Some Jew is shrewd," the Predicate is "Jew-that-is-shrewd," and the Subject is *Something*, while in the proposition "Every Christian is meek," the Predicate is "Either not Christian or else meek," while the Subject is *Anything*; unless, indeed, we find reason to prefer to say that this Proposition means, "It is false to say that a person is Christian of whom it is false to say that he is meek." In this last mode of analysis, when a Singular Subject is not in question (which case will be examined later,) the only Subject is *Something*. Either of these two modes of analysis quite clear the Subject from any Predicative ingredients; and at first sight, either seems quite favorable to the view that it is only the Subjects which belong to the Universes. Let us, however, consider the following two forms of propositions:

1. Any adept alchemist could produce a philosopher's stone of some kind or other,
2. There is one kind of philosopher's stone that any adept alchemist could produce.

We can express these on the principle that the Universes are receptacles of Subjects as follows:

1. The Interpreter having selected any individual he likes, and called it A, an object B can be found, such that, Either A would not be an adept alchemist, or B would be a philosopher's stone of some kind, and A could produce B.
2. Something B might be found, such that, no matter what the Interpreter might select and call A, B would be a philosopher's stone of some kind, while

either A would not be an adept alchemist, or else A could produce B.

In these forms there are two Universes, the one of individuals selected at pleasure by the interpreter of the proposition, the other of suitable objects.

I will now express the same two propositions on the principle that each Universe consists, not of Subjects, but the one of True assertions, the other of False, but each to the effect that there is something of a given description.

1. This is false: That something, P, is an adept alchemist and that this is false, that while something, S, is a philosopher's stone of some kind, P could produce S.
2. This is true: That something, S, is a philosopher's stone of some kind; and this is false, that something, P, is an adept alchemist while this is false, that P could produce S.

Here, the whole proposition is mostly made up of the truth or falsity of assertions that a thing of this or that description exists, the only conjunction being "and." That this method is highly analytic is manifest. Now since our whole intention is to produce a method for the perfect analysis of propositions, the superiority of this method over the other for our purpose is undeniable. Moreover, in order to illustrate how that other might lead to false logic, I will tack the predicate of No. 2, in its objectionable form upon the subject of No. 1 in the same form, and *vice versa*. I shall thus obtain two propositions which that method represents as being as simple as are Nos. 1 and 2. We shall see whether they are so. Here they are:

3. The Interpreter having designated any object to be called A, an object B may be found such that

B is a philosopher's stone of some kind, while either A is not an adept alchemist or else A could produce B.



4. Something, B, may be found, such that, no matter what the interpreter may select, and call A,

Either A would not be an adept alchemist, or B would be a philosopher's stone of some kind, and A could produce B.

Proposition 3 may be expressed in ordinary language thus: There is a kind of philosopher's stone, and if there be any adept alchemist, he could produce a philosopher's stone of some kind. That is, No. 3 differs from No. 1 only in adding that there is a kind of philosopher's stone. It differs from No. 2 in not saying that any two adepts could produce the same kind of stone, (nor that any adept could produce any existing kind,) while No. 2 asserts that some kind is both existent and could be made by every adept.

Proposition 4, in ordinary language, is: If there be (or were) an adept alchemist, there is (or would be) a kind of philosopher's stone that any adept could produce. This asserts the substance of No. 2, but only conditionally upon the existence of an adept; but it asserts, what No. 1 does not, that all adepts could produce some one kind of stone, and this is precisely the difference between No. 4 and No. 1.

To me it seems plain that the propositions 3 and 4 are both less simple than No. 1 and less simple than No. 2, each adding some thing to one of the pair first given and asserting the other conditionally. Yet the method of treating the Universes as receptacles for the metaphysical Subjects only, involves as a consequence the representation of 3 and 4 as quite on a par with 1 and 2.

It remains to show that the other method does not carry this error with it. It is the states of things affirmed or denied that are contained in the universes, then, the propositions become as follows:

3. This is true: that there is a philosopher's stone of some kind, S, and that it is false that there is an

adept, A, and that it is false that A could produce a philosopher's stone of some kind, S'. [Where it is neither asserted nor denied that S and S' are the same, thus distinguishing this from 2.]

4. This is false: That there is an adept, A, and that this is false: That there is a stone of a kind, S, and this is false: That there is an adept, A', and that this is false: That A' could produce a stone of the kind S. [Where again it is neither asserted nor denied that A and A' are identical, but the point is that this proposition holds even if they are not identical, thus distinguishing this from 1.]

These forms exhibit the greater complexity of Propositions 3 and 4, by showing that they really relate to *three* individuals each; that is to say, 3 to two possible different kinds of stone, as well as to an adept; and 4 to two possible different adepts, and to a kind of stone. Indeed, the two forms of statement of 3 and 4 on the other theory of the universes are absolutely identical in meaning with the following different forms on the same theory. Now it is, to say the least, a serious fault in a method of analysis that it can yield two analyses so different of one and the same compound.

3. An object, B, can be found, such that whatever object the interpreter may select and call A, an object, B', can thereupon be found such that B is an existing kind of philosopher's stone, and either A would not be an adept or else B' is a kind of philosopher's stone such as A could produce.
4. Whatever individual the Interpreter may choose to call A, an object, B, may be found, such that whatever individual the Interpreter may choose to call A', Either A is not an adept or B is an existing kind of philosopher's stone, and either A' is not an adept or else A' could produce a stone of the kind B.

But while my forms are perfectly analytic, the need of diagrams to exhibit their meaning to the eye (better than merely giving a separate line to every proposition said to be false,) is painfully obtrusive.\*

I will now say a few words about what you have called Categories, but for which I prefer the designation Predicaments, and which you have explained as predicates of predicates. That wonderful operation of hypostatic abstraction by which we seem to create *entia rationis* that are, nevertheless, sometimes real, furnish us the means of turning predicates from being signs that we think or think *through*, into being subjects thought of. We thus think of the thought-sign itself, making it the object of another thought-sign. Thereupon, we can repeat the operation of hypostatic abstraction, and from these second intentions derive third intentions. Does this series proceed endlessly? I think not. What then are the characters of its different members? My thoughts on this subject are not yet harvested. I will only say that the subject concerns Logic, but that the divisions so obtained must not be confounded with the different Modes of Being; Actuality, Possibility, Destiny [or Freedom from Destiny]. On the contrary, the succession of Predicates of Predicates is different in the different Modes of Being. Meantime, it will be proper that in our system of diagrammatization we should provide for the division, whenever needed, of each of our three Universes of modes of reality into *Realms* for the different Predicaments.

All the various meanings of the word "Mind," Logical, Metaphysical, and Psychological, are apt to be confounded more or less, partly because considerable logical acumen is required to distinguish some of them, and because of

\* In correcting the proofs, a good while after the above was written, I am obliged to confess that in some places the reasoning is erroneous; and a much simpler argument would have supported the same conclusion more justly; though some weight ought to be accorded to my argument here, on the whole.

the lack of any machinery to support the thought in doing so, partly because they are so many, and partly because (owing to these causes,) they are all called by one word, "mind." In one of the narrowest and most concrete of its logical meanings, a Mind is that Seme of The Truth, whose determinations become Immediate Interpretants of all other Signs whose Dynamical Interpretants are dynamically connected. In our Diagram the same thing which represents The Truth must be regarded as in another way representing the Mind, and indeed, as being the Quasi-mind of all the Signs represented on the Diagram. For any set of Signs which are so connected that a complex of two of them can have one interpretant, must be Determinations of one Sign which is a *Quasi-mind*.

Thought is not necessarily connected with a brain. It appears in the work of bees, of crystals, and throughout the purely physical world; and one can no more deny that it is really there, than that the colors, the shapes, etc. of objects are really there. Consistently adhere to that unwarrantable denial, and you will be driven to some form of idealistic nominalism akin to Fichte's. Not only is thought in the organic world, but it develops there. But as there cannot be a General without Instances embodying it, so there cannot be thought without Signs. We must here give "Sign" a very wide sense, no doubt, but not too wide a sense to come within our definition. Admitting that connected Signs must have a Quasi-mind, it may further be declared that there can be no isolated sign. Moreover, signs require at least two Quasi-minds; a *Quasi-utterer* and a *Quasi-interpreter*; and although these two are at one (i. e. are one mind) in the sign itself, they must nevertheless be distinct. In the Sign they are, so to say, *welded*. Accordingly, it is not merely a fact of human Psychology, but a necessity of Logic, that every logical evolution of thought should be dialogic. You may say that all this is

loose talk; and I admit that, as it stands, it has a large infusion of arbitrariness. It might be filled out with argument so as to remove the greater part of this fault; but in the first place, such an expansion would require a volume,—and an uninviting one; and in the second place, what I have been saying is only to be applied to a slight determination of our system of diagrammatization, which it will only slightly affect; so that, should it be incorrect, the utmost *certain* effect will be a danger that our system *may* not represent every variety of non-human thought.

There now seems to remain no reason why we should not proceed forthwith to formulate and agree upon

#### THE CONVENTIONS

##### DETERMINING THE FORMS AND INTERPRETATIONS OF

#### Existential Graphs.

##### *Convention the First: Of the Agency of the Scripture.*

We are to imagine that two parties\* collaborate in composing a PHEME, and in operating upon this so as to develop a Delome. [Provision shall be made in these Conventions for expressing every kind of PHEME as a Graph;† and it is certain that the Method could be applied to aid the development and analysis of any kind of purposive thought. But hitherto no Graphs have been studied but such as are Propositions; so that, in the resulting uncertainty as to what modifications of the Conventions might be required for other applications, they have mostly been here stated as if they were only applicable to the expression of PHEMES and the working out of necessary conclusions.]

The two collaborating parties shall be called the *Graphist* and the *Interpreter*. The Graphist shall responsibly scribe each original Graph and each addition to it, with the proper indications of the Modality to be attached to

\* They may be two bodies of persons, two persons, or two mental attitudes or states of one person.

† A Graph has already been defined on p. 503 *et seq.*

it the *relative Quality*\* of its position, and every particular of its dependence on and connections with other graphs. The Interpreter is to make such erasures and insertions of the Graph delivered to him by the Graphist as may accord with the "*General Permissions*" deducible from the Conventions and with his own purposes.

*Convention the Second: Of the Matter of the Scripture, and the Modality of the Phemes expressed.* The matter which the Graph-instances are to determine, and which thereby becomes the *Quasi-mind* in which the Graphist

\* The traditional and ancient use of the term propositional *Quality* makes it an affair of the mode of expression solely. For "Socrates is mortal" and "Socrates is immortal" are equally Affirmative, "Socrates is not mortal" and "Socrates is not immortal" are equally Negative, provided "is not" translates *non est*. If, however, "is not" is in Latin *est non*, with no difference of meaning, the proposition is infinitated. Without anything but the merest verbiage to support the supposition that there is any corresponding distinction between different meanings of propositions, Kant insisted on raising the difference of expression to the dignity of a category. In *The Monist*, Vol. VII, p. 209, I gave some reason for considering a relative proposition to be affirmative or negative according as it does or does not unconditionally assert the existence of an indefinite subject. Although at the time of writing that, nine and a half years ago, I was constrained against my inclinations, to make that statement, yet I never heartily embraced that view, and dismissed it from my mind, until after I had drawn up the present statement of the Conventions of Existential Graphs, I found, quite to my surprise, that I had herein taken substantially the same view. That is to say, although I herein speak only of "relative" quality, calling the assertion of any proposition the Affirmation of it, and regarding the denial of it as an assertion *concerning* that proposition as subject, namely, that it is false; which is my distinction of Quality Relative to the proposition either itself Affirmed, or of which the falsity is affirmed, if the Relative Quality of it is Negative, yet since every Graph in itself either recognizes the existence of a familiar Singular subject or asserts something of an indefinite subject asserted to exist in some Universe, it follows that every relatively Affirmative Graph unconditionally asserts or recognizes the occurrence of some description of object in some Universe; while no relatively Negative Graph does this. The logic of a Limited Universe of Marks suggests a different view of Quality, but careful analysis shows that it is in no fundamental conflict with the above.

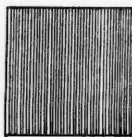
A question not altogether foreign to the subject of Quality is whether Quality and Modality are of the same general nature. In selecting a mode of representing Modality, which I have not done without much experimentation, I have finally resorted to one which commits itself as little as possible to any particular theory of the nature of Modality, although there are undeniable objections to such a course. If any particular analysis of Modality had appeared to me to be quite evident, I should have endeavored to exhibit it unequivocally. Meantime, my opinion is that the Universe is a Subject of every Proposition, and that any Modality shown by its indefiniteness to be Affirmative, such as Possibility and Intention, is a special determination of the Universe of The Truth. Something of this sort is seen in Negation. For if we say of a Man that he is not sinless, we represent the sinless as having a place only in an ideal universe which, or the part of which that contains the imagined sinless being, we then positively sever from the identity of the man in question.

and Interpreter are at one, being a Seme of *The Truth*, that is, of the widest Universe of Reality, and at the same time, a PHEME of all that is tacitly taken for granted between the Graphist and Interpreter, from the outset of their discussion, shall be a sheet, called the *PHEMIC SHEET*, upon which signs can be scribed, and from which any that are already scribed in any manner (even though they be

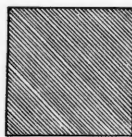
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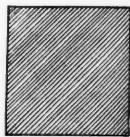
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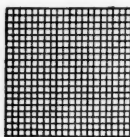


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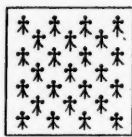


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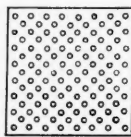
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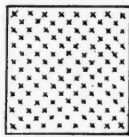
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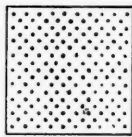


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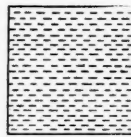
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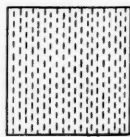
ARGENT.



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PLOMB.

Fig. I.

incised) can be erased. But certain parts of other sheets not having the significance of the PHEMIC sheet, but on which Graphs can be scribed and erased, shall be sometimes inserted in the PHEMIC sheet and exposed to view, as the Third Convention shall show. Every part of the exposed surface shall be tintured in one or another of

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twelve tinctures. These are divided into three *classes* of four tinctures each, the class-characters being called *Modes of Tincture*, or severally, Color, Fur, and Metal. The tinctures of Colour are Azure, Gules, Vert, and Purpure. Those of Fur are Sable, Ermine, Vair, and Potent. Those of Metal are Argent, Or, Fer, and Plomb. The Tinctures will in practice be represented as in Fig. I.\* The whole of any *continuous* part of the exposed surface in one tincture shall be termed a *Province*. The border of the sheet has one tincture all round; and we may imagine that it was chosen from among twelve, in agreement between the Graphist and the Interpreter at the outset. The province of the border may be called the *March*. Provinces adjacent to the March are to be regarded as overlying it; Provinces adjacent to those Provinces, but not to the March, are to be regarded as overlying the provinces adjacent to the March, and so on. We are to imagine that the Graphist always finds provinces where he needs them.

When any representation of a state of things consisting in the applicability of a given description to an individual or limited set of individuals otherwise indesignate is scribed, the Mode of Tincture of the province on which it is scribed shows whether the Mode of Being which is to be affirmatively or negatively attributed to the state of things described is to be that of Possibility, when Color will be used; or that of Intention, indicated by Fur; or that of Actuality shown by Metal. Special understandings may determine special tinctures to refer to special varieties of the three genera of Modality. Finally, the Mode of Tincture of the March may determine whether the Entire Graph is to be understood as Interrogative, Imperative, or Indicative.

\*It is chiefly for the sake of these convenient and familiar modes of representation of Petrosancta, that a modification of heraldic tinctures has been adopted. Vair and Potent here receive less decorative and pictorial Symbols. Fer and Plomb are selected to fill out the quaternion of metals on account of their monosyllabic names.

*Convention the Third: Of Areas enclosed within, but severed from, the Phemic Sheet.* The Phemic Sheet is to be imagined as lying on the smoother of the two surfaces or sides of a *Leaf*, this side being called the *recto*, and to consist of so much of this side as is continuous with the March. Other parts of the *recto* may be *exposed* to view. Every Graph-instance on the Phemic Sheet is posited unconditionally (unless, according to an agreement between Graphist and Interpreter, the Tincture of its own Province or of the March should indicate a condition); and every Graph-instance on the *recto* is posited affirmatively and, in so far as it is indeterminate, indefinitely.

Should the Graphist desire to negative a Graph, he must scribe it on the *verso*, and then, before delivery to the Interpreter, must make an incision, called a *Cut*, through the Sheet all the way round the Graph-instance to be denied, and must then turn over the excised piece, so as to *expose* its rougher surface carrying the negatived Graph-instance. This reversal of the piece is to be conceived to be an inseparable part of the operation of making a *Cut*.\* But if the Graph to be negatived includes a *Cut*, the twice negatived Graph within that *Cut* must be scribed on the *recto*, and so forth. The part of the exposed surface that is continuous with the part just outside the *Cut* is called the *Place of the Cut*. A *Cut* is neither a Graph nor a Graph-instance; but the *Cut* together with all that it encloses exposed is termed an *Enclosure*, and is conceived to be an Instance of a Graph *scribed* on the *Place of the Cut*, which is also termed the *Place of the Enclosure*. The surface within the *Cut*, continuous with the parts just within it, is termed the *Area* of the *Cut* and of the *Enclosure*; and the part of the *recto* continuous with the

\* I am tempted to say that it is the reversal alone that effects the denial, the *Cut* merely cutting off the Graph within from assertion concerning the Universe to which the Phemic Sheet refers. But that is not the only possible view, and it would be rash to adopt it definitely, as yet.

March, (i. e., the Phemic Sheet,) is likewise termed an *Area*, namely the Area of the Border. The Copulate of all that is scribed on any one Area, including the Graphs of which the Enclosures whose Place is this Area are Instances, is called the *Entire Graph* of that Area; and any part of the Entire Graph, whether graphically connected with or disconnected from the other parts, provided it might be the Entire Graph of the Sheet, is termed a Partial Graph of the Area.

There may be any number of Cuts, one within another, the Area of one being the Place of the next, and since the Area of each is on the side of the leaf opposite to its Place, it follows that *recto* Areas may be *exposed* which are not parts of the Phemic Sheet. Every Graph-instance on a *recto* Area is affirmatively posited, but is posited conditionally upon whatever may be signified by the Graph on the Place of the Cut of which this Area is the Area. [It follows that Graphs on Areas of different Enclosures on a *verso* Place are only alternatively affirmed, and that while only the Entire Graph of the Area of an Enclosure on a *recto* Place is denied, but not its different Partial Graphs, except alternatively, the Entire Graphs of Areas of different Enclosures on one *recto* Place are copulatively denied.]

Every Graph-instance must lie upon one Area,\* although an Enclosure may be a part of it. Graph-instances on different Areas are not to be considered as, nor by any permissible latitude of speech to be called, Parts of one Graph-instance, nor Instances of Parts of one Graph; for it is only Graph-instances on one Area that are called Parts of one Graph-instance, and that only of a Graph-instance

\* For, of course, the Graph-instance must be on one sheet; and if part were on the *recto*, and part on the *verso*, it would not be on one continuous sheet. On the other hand, a Graph-instance can perfectly well extend from one Province to another, and even from one *Realm* (or space having one Mode of Tincture) to another. Thus, the Spot, "—is in the relation—to—," may, if the relation is that of an existent object to its purpose, have the first Peg on Metal, the second on Color, and the third on Fur.

on that same Area; for though the Entire Graph on the Area of an enclosure is termed the *Graph of the Enclosure*, it is no Part of the Enclosure and is connected with it only through a denial.

*Convention the Fourth: concerning Signs of Individuals and of Individual Identity.* A single dot, not too minute, or single congeries of contiguous pretty large dots, whether in the form of a line or surface, when placed on any exposed Area, will refer to a single member of the Universe to which the Tincture of that Area refers, but will not thereby be made to refer determinately to any one. But do not forget that separate dots, or separate aggregates of dots, will not necessarily denote different Objects.

By a *rheme*, or *predicate*, will here be meant a blank form of proposition which might have resulted by striking out certain parts of a proposition, and leaving a *blank* in the place of each, the parts stricken out being such that if each blank were filled with a proper name, a proposition (however nonsensical) would thereby be recomposed. An ordinary predicate of which no analysis is intended to be represented will usually be *written* in abbreviated form, but having a particular point on the periphery of the written form appropriated to each of the blanks that might be filled with a proper name. Such written form with the appropriated points shall be termed a *Spot*; and each appropriated point of its periphery shall be called a *Peg* of the Spot. If a heavy dot is placed at each Peg, the Spot will become a Graph expressing a proposition in which every blank is filled by a word (or concept) denoting an indefinite individual object, "something."

A heavy line shall be considered as a continuum of contiguous dots; and since contiguous dots denote a single individual, such a line without any point of branching will signify the identity of the individuals denoted by its extremities, and the type of such unbranching line shall

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be the Graph of Identity, any instance of which (on one area, as every Graph-instance must be,) shall be called a *Line of Identity*. The type of a three-way point of such a line (Fig. 2) shall be the *Graph of Teridentity*; and it shall be considered as composed of three contiguous Pegs of a Spot of Identity. An extremity of a Line of Identity not abutting upon another such Line in another area shall be called a *Loose End*. A heavy line, whether confined to one area or not (and therefore not generally being a Graph-instance,) of which two extremities abut upon pegs of spots shall be called a *Ligature*. Two lines cannot abut upon the same peg other than a point of teridentity. [The purpose of this rule is to force the recognition of the demonstrable logical truth that the concept of teridentity is not mere identity. It is identity *and* identity, but this

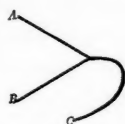


Fig. 2.



Fig. 3.

"and" is a distinct concept, and is precisely that of teridentity.] A Ligature crossing a Cut is to be interpreted as unchanged in meaning by erasing the part that crosses to the Cut and attaching to the two Loose Ends so produced two Instances of a Proper Name nowhere else used; such a Proper name (for which a capital letter will serve,) being termed a *Selective*. In the interpretation of Selectives it is often necessary to observe the rule which holds throughout the System, that the Interpretation of Existential Graphs must be *endoporeutic*, that is, the application of a Graph on the Area of a Cut will depend on the predetermination of the application of that which is on the Plate of the Cut.

In order to avoid the intersection of Lines of Identity,

either a Selective may be employed, or a *Bridge*, which is imagined to be a bit of paper ribbon, but will in practice be pictured as in Fig. 3.

*Convention the Fifth: Of the Connections of Graph-Instances.* Two partial Graph-Instances are said to be *individually and directly connected*, if, and only if, in the Entire Graph, one individually is, either unconditionally or under some condition, and whether affirmatively or negatively, made a Subject of both. Two Graph-Instances connected by a ligature are explicitly and definitely individually and directly connected. Two Graph-Instances in the same Province are thereby explicitly, although indefinitely, individually and directly connected, since both, or one and the negative of the other, or the negative of both, are asserted to be true or false together, that is, under the same circumstances, although these circumstances are not formally defined, but are left to be interpreted according to the nature of the case. Two Graph-instances not in the same Province, though on the same Mode of Tincture are only in so far connected that both are in the same Universe. Two Graph-Instances in different Modes of Tincture are only in so far connected that both, or one and the negative of the other, or the negative of both, are posited as appertaining to the Truth. They cannot be said to have any individual and direct connection. Two Graph-instances that are not individually connected within the innermost Cut which contains them both cannot be so connected at all; and every ligature connecting them is meaningless and may be made or broken.

Relations which do not imply the occurrence in their several universes of all their correlates must not be expressed by Spots or single Graphs,\* but all such relations can be expressed in the System.

\* It is permissible to have such spots as "possesses the character," "is in the real relation to," but it is not permissible to have such a spot as "can prevent the existence of."

I will now proceed to give a few examples of Existential Graphs in order to illustrate the method of interpretation, and also the *Permissions of Illative Transformation* of them.

If you carefully examine the above conventions, you will find that they are simply the development, and excepting in their insignificant details, the inevitable result of the development of the one convention that if any Graph, A, asserts one state of things to be real and if another graph, B, asserts the same of another state of things, then AB, which results from setting both A and B upon the sheet, shall assert that both states of things are real. This was not the case with my first system of Graphs, described in Vol. VII of *The Monist*, which I now call *Entitative Graphs*. But I was forced to this principle by a series of considerations which ultimately arrayed themselves into an exact logical deduction of all the features of Existential Graphs which do not involve the Tinctures. I have no room for this here; but I state some of the points arrived at somewhat in the order in which they first presented themselves.

In the first place, the most perfectly analytical system of representing propositions must enable us to separate illative transformations into indecomposable parts. Hence, an illative transformation from any proposition, A, to any other, B, must in such a system consist in first transforming A into AB, followed by the transformation of AB into B. For an omission and an insertion appear to be indecomposable transformations and the only indecomposable transformations. That is, if A can be transformed by insertion into AB, and AB by omission in B, the transformation of A into B can be decomposed into an insertion and an omission. Accordingly, since logic has primarily in view argument, and since the conclusiveness of an argument can never be weakened by adding to the premisses



nor by subtracting from the conclusion, I thought I ought to take the general form of argument as the basal form of composition of signs in my diagrammatization; and this necessarily took the form of a "scroll," that is (See Figs. 4, 5, 6) a curved line without contrary flexure and returning into itself after once crossing itself, and thus forming an outer and an inner "close." I shall call the outer boundary the *Wall*; and the inner, the *Fence*. In the outer I scribed the Antecedent, in the inner the Consequent, of a Conditional Proposition *de inesse*. The scroll was not taken for this purpose at hap-hazard, but was the result of experiments and reasonings by which I was brought to see that it afforded the most faithful Diagram of such a Proposition. This form once obtained, the logically inevitable

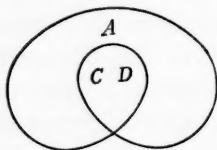


Fig. 4.

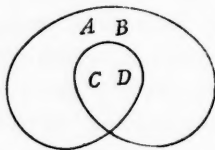


Fig. 5.

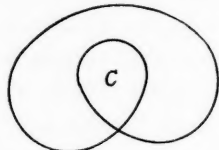


Fig. 6.

development brought me speedily to the System of Existential Graphs. Namely, the idea of the scroll was that Fig. 4, for example, should assert that if A be true (under the actual circumstances), then C and D are both true. This justifies Fig. 5, that if both A and B are true, then both C and D are true, no matter what B may assert, any insertion being permitted in the outer close, and any omission from the inner close. By applying the former clause of this rule to Fig. 6, we see that this scroll with the outer close void, justifies the assertion that if no matter what be true, C is in any case true; so that the two walls of the scroll, when nothing is between them, fall together, collapse, disappear, and leave only the contents of the inner close standing, asserted, in the open field. Suppos-

ing, then, that the contents of the inner scroll had been CD, these would have been left standing, both asserted; and we thus return to the principle that writing assertions together on the open sheet asserts them all. Now, Reader, if you will just take pencil and paper and scribe the scroll expressing that if A be true, then it is true that if B be true C and D are true, and compare this with Fig. 5, which amounts to the same thing in meaning, you will see that scroll walls with a void between them collapse even when they belong to different scrolls; and you will further see that a scroll is really nothing but one oval within another. Since a Conditional *de inesse* (unlike other conditionals,) only asserts that either the antecedent is false or the consequent is true, it all but follows that if

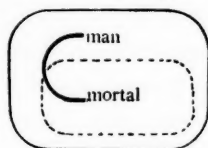


Fig. 7.



Fig. 8.

the latter alternative be suppressed by scribing nothing but the antecedent, which may be any proposition, in an oval, that antecedent is thereby denied. The use of a heavy line as a juncture signifying identity is inevitable; and since Fig. 7 must mean that if anything is a man, it is mortal, it will follow that Fig. 8 must mean "Something is a man."

The first permission of illative transformation is now evident as follows:

*First Permission, called "The Rule of Deletion and Insertion."* Any Graph-Instance can be deleted from any recto Area, (including the severing of any Line of Identity,) and any Graph-instance can be inserted on any verso Area, (including as a Graph-instance the juncture of any two Lines of Identity or Points of Teridentity.)

The justice of the following will be seen instantly by students of any form of Logical Algebra, and with very little difficulty by others:

*Second Permission, called "The Rule of Iteration and Deiteration."* Any Graph scribed on any Area may be Iterated in or (if already Iterated,) may be Deiterated by a deletion from that Area or from any other Area included within that. This involves the Permission to distort a line of Identity, at will.

To iterate a Graph means to scribe it again, while joining by Ligatures every Peg of the new Instance to the corresponding Peg of the Original Instance. To deiterate a Graph is to erase a second Instance of it, of which each Peg is joined by a Ligature to a first Instance of it. One Area is said to be *included within* another if, and only if, it is the Area of a Cut whose Place either is that Area or else, is an Area which, according to this definition, must be regarded as *included within* that other. By this Permission, Fig. 9 may be transformed into Fig. 10, and thence, by Permission No. 1, into Fig. 11.

We now come to the Third Permission, which I shall state in a form which is valid, sufficient for its purpose, and convenient in practice, but which cannot be assumed as an undeduced Permission, for the reason that it allows us to regard the Inner Scroll, after the Scroll is removed, as being a part of the Area on which the Scroll lies. Now this is not strictly either an Insertion or a Deletion; and a perfectly analytical System of Permissions should permit only the indecomposable operations of Insertion and Deletion of Graphs that are simple in expression. The more scientific way would be to substitute for the Second and Third Permissions the following Permission:

*If an Area,  $\Upsilon$ , and an Area,  $\Omega$ , be related in any of these four ways, viz., (1) If  $\Upsilon$  and  $\Omega$  are the same Area; (2) If  $\Omega$  is the Area of an Enclosure whose Place is  $\Upsilon$ ; (3) If  $\Omega$  is*

*the Area of an Enclosure whose Place is the Area of a second Enclosure whose Place is  $\Upsilon$ ; or (4) If  $\Omega$  is the Place of an Enclosure whose Area is vacant except that it is the Place of an Enclosure whose Area is  $\Upsilon$ , and except that it may contain ligatures, identifying Pegs in  $\Omega$  with Pegs in  $\Upsilon$ ; then, if  $\Omega$  be a recto area, any simple Graph already scribed upon  $\Upsilon$  may be iterated upon  $\Omega$ ; while if  $\Omega$  be a verso Area, any simple Graph already scribed upon  $\Upsilon$  and iterated upon  $\Omega$  may be deiterated by being deleted or abolished from  $\Omega$ .*

These two Rules (of Deletion and Insertion, and of Iteration and Deiteration) are substantially all the undeduced Permissions needed; the others being either Consequences or Explanations of these. Only, in order that this may be true, it is necessary to assume that all indemonstrable implications of the Blank have from the beginning been scribed upon distant parts of the Phemic Sheet, upon any part of which they may, therefore, be iterated at will. I will give no list of these implications, since it could serve no other purpose than that of warning beginners that necessary propositions not included therein were deducible from the other permissions. I will simply notice two principles the neglect of which might lead to difficulties. One of these is that it is physically impossible to delete or otherwise get rid of a Blank in any Area that contains a Blank, whether alone or along with other Graph-Instances. We may, however, assume that there is one Graph, and only one, an Instance of which entirely fills up an Area, without any Blank. The other principle is that, since a Dot merely asserts that some individual object exists, and is thus one of the implications of the Blank, it may be inserted in any Area; and since the Dot will signify the same thing whatever its size, it may be regarded as an Enclosure whose Area is filled with an Instance of that sole Graph that excludes the Blank. The

Dot, then, denies that Graph, which may, therefore, be understood as the absurd Graph, and its signification may be formulated as "Whatever you please is true." The absurd Graph may also take the form of an Enclosure with its Area entirely Blank, or enclosing only some Instance of a Graph implied in the Blank. These two principles will enable the Graphist to thread his way through some Transformations which might otherwise appear paradoxical and absurd.

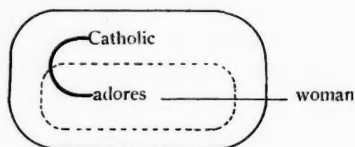


Fig. 9.

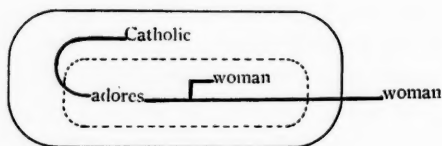


Fig. 10.

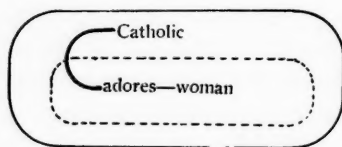


Fig. 11.

*Third Permission; called "The Rule of the Double Cut." Two Cuts one within another, with nothing between them, unless it be Ligatures passing from outside the outer Cut to inside the inner one, may be made or abolished on any Area.*

Let us now consider the Interpretation of such Ligatures. For that purpose, I first note that the Entire Graph of any *recto* Area is a wholly particular and affirmative

Proposition or Copulation of such Propositions. By "wholly particular," I mean, having for every Subject an indesignate individual. The Entire Graph of any *verso* Area is a wholly universal negative proposition or a disjunction of such propositions.

The first time one hears a Proper Name pronounced, it is but a name, predicated, as one usually gathers, of an existent, or at least historically existent, individual object, of which, or of whom, one almost always gathers some additional information. The next time one hears the name, it is by so much the more definite; and almost every time one hears the name, one gains in familiarity with the object. A Selective is a Proper Name met with by the Interpreter for the first time. But it always occurs twice, and usually on different areas. Now the Interpretation, by Convention No. 3, is to be Endoporeutic, so that it is the outermost occurrence of the Name that is the earliest.

Let us now analyze the interpretation of a Ligature passing through a Cut. Take, for example, the Graph of Fig. 12. The partial Graph on the Place of the Cut

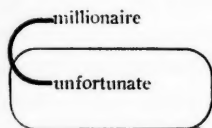


Fig. 12.

asserts that there exists an individual denoted by the extremity of the line of identity on the Cut, which is a millionaire. Call that individual C. Then, since contiguous dots denote the same individual objects, the extremity of

the line of identity on the Area of the cut is also C, and the Partial Graph on that Area, asserts that, let the Interpreter choose whatever individual he will, that individual is either not C, or else is not unfortunate. Thus, the Entire Graph asserts that there exists a millionaire who is not unfortunate. Furthermore, the Enclosure lying in the same Argent Province as the "millionaire," it is asserted that this individual's being a millionaire is connected with his not being unfortunate. This example

shows that the Graphist is permitted to extend any Line of Identity on a recto Area so as to carry an end of it to any Cut in that area. Let us next interpret Fig. 13. It

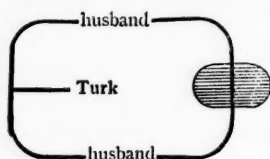


Fig. 13

obviously asserts that there exists a Turk who is at once the husband of an Individual denoted by a point on the Cut, which individual we may name U, and is the husband of an Individual, whom we may name V, denoted by another point on the Cut. And the Graph on the Area of the cut, declares that whatever Individual the Interpreter may select either is not, and cannot be, U or is not and cannot be V. Thus, the Entire Graph asserts that there is an existent Turk who is husband of two existent persons; and the "husband," the "Turk" and the enclosure, all being in the same Argent province, although the Area of the Enclosure is on color, and thus denies the possibility of the identity of U and V, all four predications are true together, that is, are true under the same circumstances, which circumstances should be defined by a special convention when anything may turn upon what they are. For the sake of illustrating this, I shall now scribe Fig. 14 all in one province. This may

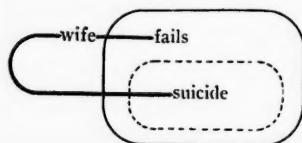


Fig. 14.

be read, "There is some married woman who will commit suicide in case her husband fails in business." This evidently goes far beyond saying that if every married man fails in business some married woman will commit suicide. Yet note that since the Graph is on Metal it asserts a conditional proposition *de inesse* and only means that there is a married woman whose husband does not fail or else she commits suicide. That, at least, is all it will seem to



mean if we fail to take account of the fact, that being all in one Province, it is said that her suicide is *connected* with his failure. Neglecting that, the proposition only denies that every married man fails, while no married woman commits suicide. The logical principle is that to say that there is some one individual of which one or other of two predicates is true is no more than to say that there either is some individual of which one is true or else there is some individual of which the other is true. Or, to state the matter as an illative permission of the System of Existential Graphs,

*Fourth Permission. If the smallest Cut which wholly contains a Ligature connecting two Graphs in different Provinces has its Area on the side of the Leaf opposite to that of the Area of the smallest Cut that contains those two Graphs, then such Ligature may be made or broken at pleasure, as far as these two Graphs are concerned.*

Another somewhat curious problem concerning ligatures is to say by what principle it is true, as it evidently is true that the passage of ligatures from without the outer of two Cuts to within the inner of them will not prevent the two from collapsing in case there is no other Graph instance between them. A little study suffices to show that this may depend upon the ligatures' being replaceable by Selectives where they cross the Cuts, and that a Selective is always, at its first occurrence, a new predicate. For it is a principle of Logic that in introducing a new predicate one has a right to assert what one likes concerning it, without any restriction, as long as one implies no assertion concerning anything else. I will leave it to you, Reader, to find out how this principle accounts for the collapse of the two Cuts. Another solution of this problem, not depending on the superfluous device of Selectives is afforded by the second enunciation of the Rule of Iteration and Deiteration; since this permits the Graph of the Inner

Close to be at once iterated on the Phemic Sheet. One may choose between these two methods of solution.

The System of Existential Graphs which I have now sufficiently described,—or, at any rate, have described as well as I know how, leaving the further perfection of it to others,—greatly facilitates the solution of problems of Logic, as will be seen in the sequel, not by any mysterious properties, but simply by substituting for the symbols in which such problems present themselves, concrete visual figures concerning which we have merely to say whether

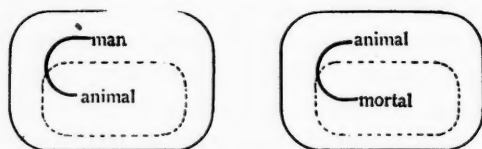


Fig. 15.

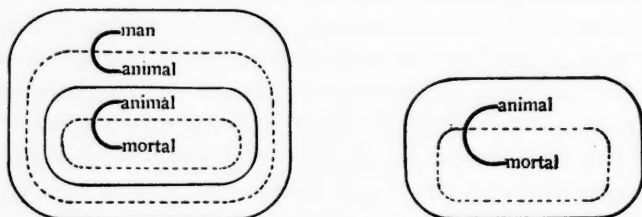


Fig. 16.

or not they admit certain describable relations of their parts. Diagrammatic reasoning is the only really fertile reasoning. If logicians would only embrace this method, we should no longer see attempts to base their science on the fragile foundations of metaphysics or a psychology not based on logical theory; and there would soon be such an advance in logic that every science would feel the benefit of it.

This System may, of course, be applied to the analysis of reasonings. Thus, to separate the syllogistic illation,

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"Any man would be an animal, and any animal would be mortal; therefore, any man would be mortal," the Premises are first scribed as in Fig. 15. Then by the rule of Iteration, a first illative transformation gives Fig. 16. Next, by the permission to erase from a recto Area, a

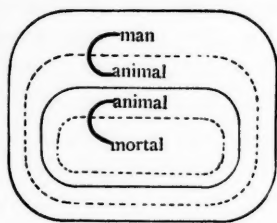


Fig. 17.

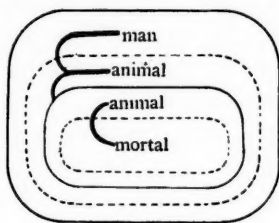


Fig. 18.

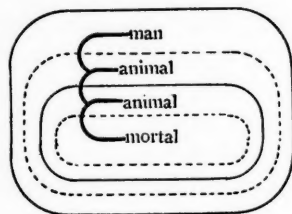


Fig. 19.

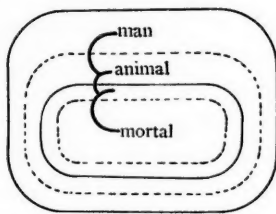


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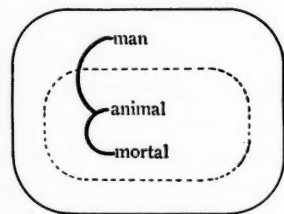


Fig. 21.

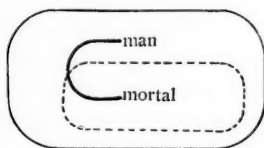


Fig. 22.

second step gives Fig. 17. Then, by the permission to deform a line of Identity on a recto Area, a third step gives Fig. 18. Next, by the permission to insert in a verso Area, a fourth step gives Fig. 19. Next, by Deiteration, a fifth step gives Fig. 20. Next, by the collapse of

two Cuts, a sixth step gives Fig. 21; and finally, by omission from a recto Area, a seventh step gives the conclusion Fig. 22. The analysis might have been carried a little further, by means of the Rule of Iteration and Deiteration, so as to increase the number of distinct inferential steps to nine, showing how complex a process the drawing of a syllogistic conclusion really is. On the other hand, it need scarcely be said that there are a number of *deduced* liberties of transformation, by which even much more complicated inferences than a syllogism can be performed at a stroke. For that sort of problem, however, which consists in drawing a conclusion or assuring oneself of its correctness, this System is not particularly adapted. Its true utility is in the assistance it renders,—the support to the mind, by furnishing concrete diagrams upon which to experiment,—in the solution of the most difficult problems of logical theory.

I mentioned on an early page of this paper that this System leads to a different conception of the Proposition and Argument from the traditional view that a Proposition is composed of Names, and that an Argument is composed of Propositions. It is a matter of insignificant detail whether the term Argument be taken in the sense of the Middle Term, in that of the Copulate of Premisses, in that of the setting forth of Premisses and Conclusion, or in that of the representation that the real facts which the premisses assert (together, it may be, with the mode in which those facts have come to light) logically signify the truth of the Conclusion. In any case, when an Argument is brought before us, there is brought to our notice (what appears so clearly in the Illative Transformations of Graphs) a process whereby the Premisses bring forth the Conclusion, not informing the Interpreter of its Truth, but appealing to him to assent thereto. This Process of Transformation, which is evidently the kernel of the mat-

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ter, is no more built out of Propositions than a motion is built out of positions. The logical relation of the Conclusion to the Premisses might be asserted; but that would not be an Argument, which is essentially intended to be understood as representing what it represents only in virtue of the logical habit which would bring any logical Interpreter to assent to it. We may express this by saying that the Final (or quasi-intended) Interpretant of an Argument represents it as representing its Object after the manner of a Symbol. In an analogous way the relation of Predicate to Subject which is *stated* in a Proposition might be merely described in a Term. But the essence of the Proposition is that it intends, as it were, to be regarded as in an existential relation to its Object, as an Index is, so that its assertion shall be regarded as evidence of the fact. It appears to me that an assertion and a command do not differ essentially in the nature of their Final Interpretants as in their Immediate, and so far as they are effective, in their Dynamical Interpretants; but that is of secondary interest. The Name, or any Seme, is merely a substitute for its Object in one or another capacity in which respect it is all one with the Object. Its Final Interpretant thus represents it as representing its Object after the manner of an Icon, by mere agreement in idea. It thus appears that the difference between the Term, the Proposition, and the Argument, is by no means a difference of complexity, and does not so much consist in structure as in the services they are severally intended to perform.

For that reason, the ways in which Terms and Arguments can be compounded cannot differ greatly from the ways in which Propositions can be compounded. A mystery, or paradox, has always overhung the question of the Composition of Concepts. Namely, if two concepts, A and B, are to be compounded, their composition would

seem to be necessarily a third ingredient Concept, C, and the same difficulty will arise as to the Composition of A and C. But the Method of Existential Graphs solves this riddle instantly by showing that, as far as propositions go, and it must evidently be the same with Terms and Arguments, there is but one general way in which their Composition can possibly take place; namely, each component must be indeterminate in some respect or another; and in their composition each determines the other. On the *recto* this is obvious: "Some man is rich" is composed of "Something is a man" and "something is rich," and the two somethings merely explain each other's vagueness in a measure. Two simultaneous independent assertions are still connected in the same manner; for each is in itself vague as to the Universe or the "Province" in which its truth lies, and the two somewhat define each other in this respect. The composition of a Conditional Proposition is to be explained in the same way. The Antecedent is a Sign which is Indefinite as to its Interpretant; the Consequent is a Sign which is Indefinite as to its Object. They supply each the other's lack. Of course, the explanation of the structure of the Conditional gives the explanation of negation; for the negative is simply that from whose Truth it would be true to say that anything you please would follow *de inesse*.

In my next paper, the utility of this diagrammatization of thought in the discussion of the truth of Pragmaticism shall be made to appear.

CHARLES SANTIAGO SANDERS PEIRCE.

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## PRAGMATISM, OLD AND NEW.

THE metaphysics of pure experience, which, as Professor James expresses it, is "in the atmosphere," is regarded by its exponents as in effect a new philosophy, and results of far-reaching importance are predicted for it. Its pragmatic method and its humanistic point of view are considered as likely to work a fundamental change in our notions concerning reality. In this spirit writes Schiller when he says, "The pragmatic method in philosophy bids fair to be as serviceable as the Darwinian in biology." It is held that pragmatism has reconstructed the meaning of old terms and given them an entirely different content. It promises to reinterpret the world, not in worn-out logical formulæ and musty epistemological concepts, but by categories cast in the moulds of fresh realism and filled with vital experience. On the other hand the critics of pragmatism are accused of using world-old arguments, familiar to Zeno. They are advocates of "temple-like" systems of thought; they deal in verbal subtleties and transcendental hypotheses beyond all possibility of experimental verification or even of imagination.

There is truth in these assertions as to the novelty of the pragmatic principle, and the antiquity of the arguments brought against it, but it is only a half truth. That the pragmatic method is something entirely, or even largely, new I cannot fully convince myself. There is much that is fresh and vigorous about humanism,—the



very fact that James and Dewey are among its most zealous advocates would assure this to be the case. These writers and their followers have made many neglected facts vital, have opened up forgotten vistas of truth and emphasized discarded realities; but for all this pragmatism has in it not a little that recalls the *πάντα ῥεῖ* of Heraclitus and the *homo mensura* of Protagoras. It is not to be wondered at, then, that these ancient doctrines find responses that date in part from the centuries when philosophic thought first formulated itself into clear expression on Hellenic soil; and we need not be surprised if we again witness a conflict similar to that which idealism once waged against sophism (and here I use the word in no objectionable sense, but in its original philosophical significance); that later realism fought with nominalism; and which in modern times absolutism has carried on with empiricism. It would seem as if we were once more to attempt to decide whether human experience is self-containing and self-satisfying; or whether we are not called upon to push beyond all experience, actual and possible, to those transcendent ideals of thought and action that Kant placed at the end of his philosophy.

Under such circumstances it should be of advantage to indicate the points of similarity between humanism and similar philosophical tendencies in the history of thought, and to make a note of the replies that from time to time have been made to the contention that phenomenal reality is "the be-all and the end-all" of existence. Hence it may be of service to gather these thoughts now so much in the air, in order that their trend may be more definitely discovered and their ultimate meaning outlined.

But right here the pragmatist will doubtless object that such a procedure is an attempt to categorize and make static something that can be comprehended only by experiencing it as a living, budding, growing system of real-

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ity; but I can reply only by saying that to think means to do this very thing—that it is impossible to catch experience “on the fly,” that it must be made to an extent wooden and dead in order to be comprehended. Description; however real and exact, must always remain description, and to comprehend in any more than from a hand to mouth fashion we must go beyond description; we must bring in (sad as the necessity may be) our logical formulæ and metaphysical categories; we must “prepare” our realities if we wish to cross-section them for examination, we must stain them with the foreign matter of thought if we are to trace their windings and crossings. I know this is not pragmatic, except in so far as it is necessary, or at least useful, and then it should receive the full pragmatic sanction.

Let us then examine more closely what pragmatism means. Its opponents accuse it of subjectivism and pluralism (which the pragmatist denies with greater or less vigor) and its adherents admit its phenomenalism and make much of its utilitarianism. Its empirical and utilitarian aspects are considered to be positive merits, which the absolutists have failed to recognize as such, because of the perversity and hardness of their intellects and because of their own “comparatively slack hold on the realities of the situation.” But surely subjectivism, pluralism, phenomenalism and utilitarianism are by no means new doctrines, and for these aspects of pragmatism nothing particularly novel should be claimed. The points of view which these doctrines set forth to-day are in essence the same as they have been in the past, while the objections to them present a not fundamentally different attitude of mind in the present time than in past centuries. This the further discussion will attempt to amplify.

First, however, we must consider briefly the justice of the assertion that pragmatism is subjective and plural-

istic. The pragmatist categorically denies that he is a solipsist. Indeed James asserts that the doctrine has a more vital relation with realism than with the subjective scepticism of the English empiricists. Those who bring the accusation make much of the criterion of satisfaction employed by the pragmatist as a test of truth. Satisfaction, it is urged, is an individual matter. Royce, for example, maintains that for pragmatism truth means nothing more than that which just now meets my individual needs. To this the reply is made that the satisfaction which a situation calls forth attaches itself to something clearly objective. The individual consciousness is regarded as an element in the situation which continually develops. Professor Moore writes: "If we remember . . . that this sense of freedom and harmony\* to which we last appealed is not any bare, detached, unmediated fragment of feeling that we may happen to find, but is the issue of a process of purposing and thinking, including experimentation and verification based on results of previous thoughts and actions, involving as many individual minds and as much of the material world as you will. . . . the last suspicion of subjectivism disappears."

James also writes in a similar vein; "Humanism says that satisfactoriness is what distinguishes the true from the false. . . . But nothing is more obvious than the fact that both the object and the past experience of the object may be the very things about it that most seem satisfactory and that most incite us to believe in them."

These two statements, as well as others of a similar nature, must be interpreted as meaning that a situation is not true because it is satisfactory, but merely that the truth gives satisfaction. Truth is ultimately conditioned by that which transcends the immediate experience and leads up to it. It is much as if we should say that it is

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the high barometer which distinguishes pleasant from unpleasant weather, meaning by it only of course that the barometer is merely the indication of atmospheric states and in no sense a condition of them. In exactly the same way satisfaction on the whole accompanies those situations which we call true, but it does not make the truth, it merely registers the fact that certain relations and conditions beyond the satisfaction are in existence.

If then this is all the pragmatist intends by his statement he cannot mean in any final sense that the true is the satisfactory, but only that "which gives the maximal combinations of satisfaction." Truth and satisfaction are not the same. Satisfaction is the result of truth, the badge by which it is distinguished.

If this is all the doctrine means, I can hardly understand how any one, absolutist or empiricist, can dispute it. But in this particular pragmatism, then, it is no new discovery, or any departure from the generally accepted attitude of looking at reality. There is indeed nothing about which to dispute. No one but the most confirmed pessimist would for a moment maintain that on the whole and in the long run the false and the satisfactory coincide. I am inclined to think, however, there is in the mind of the pragmatist the feeling that in some way satisfaction is the body and essence of truth, just as according to the James-Lange theory a feeling of bodily change is the essence of emotion. If this latter interpretation is put on the assertion that truth is satisfaction then the charge of subjectivism and solipsism is difficult to disprove. Each and every state of intellectual satisfaction must then point to the truth, but this can hardly be accepted as a correct interpretation of the facts. Indeed it often seems as if the more dogmatic and ignorant the individual really is, by just so much his satisfaction in his assertions is intensified. There are less thought-crises and disturbances in the mind of such a

person than in that of the earnest searcher after truth. Who can say when measured from the standpoint of real satisfaction that the psychic calm of the unreflecting dogmatist is not on the whole greater than that of the critical investigator, and if greater then to that extent possessing more of truth?

If such a conception of truth be held to we are brought face to face with the revival of the contention of the Sophists as to the epistemological equality between desirability and intellectual reality, and we can not free such a conception from the charge of solipsism. But whether we take this narrower or the more liberal interpretation of the pragmatic statement that truth is satisfaction, we are confronted with no new doctrine.

In regard to the assertion that experience, which is the final reality for the humanist, is individual and hence subjective, the pragmatist asserts that experience as such (pure experience) is neither subject nor object, but rather the reality from which both develop. To James pure experience is "the instant field of the present," a simple *that* undifferentiated into thing and thought. It is naive immediacy retrospectively separated into two parts. In one center it plays the part of a knower, while in another it plays the part of the thing known. In one instance it figures as a thought, in another as a thing. It is the stuff from which everything is composed. Yet pure experience is not actually experienced as far as we know it as normal individuals. "Pure experience is the name which I give," says James, "to the original flux of life before reflexion has categorized it. Only new-born babes and persons in semi-coma from sleep, drugs, illness or blows can have an experience pure in the literal sense of a *that* which is not yet a definite *what*." Again, "Pure experience is but another form for feeling or sensation." Thus experience is made objective by being reduced to

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mere sensation—sensation in general which belongs to no one in particular. Experience in this sense “may be taken as a point in which several experiences join. Thus several minds may know the same object.”

It is a question whether objectivity can be secured in this way; indeed according to one notion sensation and feeling are the most subjective of all mental elements, and a philosophy which makes them the ultimate reality is in danger of ending, as did that of Locke and Hume, in complete subjectivism. It is only by the most radical reconstruction of terms that such a doctrine can be made to serve its purpose of getting beyond the individual experience. The reconstruction as it stands would seem to hold something as follows: Sensation is a general term for an experience in general, and experience of nothing in particular and by no one in particular. From this experience which leans upon nothing, which sustains itself in the infinite void, is developed a consciousness that knows, and a world that is known. It itself is not consciousness, for consciousness is a function born of experience and conditioned by it. We have then an exact reversal of the ordinary way of thinking, which holds that experience demands as its condition a subject which experiences, a consciousness which conditions such experience. It is hard for those who have been inoculated with the Cartesian fiction(?) of the *res cogitans* to think in such terms as these. The words are understood, but the meaning will not readily be grasped—indeed we have no images by which to construe such a doctrine; it is as much beyond our experience as is the absolute of the most perverse and wide-bound idealist. It would be better to avoid the use of such terms as experience, feeling and sensation in setting forth this doctrine if it is to be comprehended. A better term for it would be the Kantian  $x$ . As such there

could be no objection to it, as expressing a mystery beyond comprehension.

I do not believe, however, that the pragmatist easily thinks in such terms. It is more probable that feeling and sensation mean for him what they mean for others, namely something quite individual and particular; and that experience for him means not experience in general but experience in particular, *my experience*; and truth based on such grounds is again open to the suspicion of subjectivism. On the whole it seems by no means certain that the pragmatist has escaped the charge of subjectivism, as he himself would have us believe.

We can more briefly discuss the alleged pluralism of the pragmatist's doctrine. The conception of pure experience as set forth by Professor James may claim from one point of view the merit of being monistic, since the pure experience is the stuff of all our derived and secondary realities. It is the finality itself—the brooding unity from which emerges the self and the world. Yet, on the other hand, Professor James explicitly asserts that “all experience is a process” and that “no point of view can ever be the last one. Every one is insufficient and off its balance.” Further, “there is vastly more discontinuity in the sum total of experience than we commonly suppose” and our experiences, taken all together, are “a quasi-chaos.” Whatever may be the monistic assumptions contained in the conception of a mere experience as such from which experience as we know it develops, it is certain that the experience in which the law of pragmatism holds good must be according to these above statements no more single or abiding than the universe of Herakleitus, nor more lastingly true than the world of Protagoras. Every-day experience is pluralistic. This is entirely in accord with the views of Professor James in his conceptions of the varieties of experience.



But a word need be said in regard to the empiricism of the doctrine of pragmatism. It is epitomized in the following statement of James, "Everything real must be experienced; and everything experienced must be real."

Utilitarianism is perhaps the most striking feature of the doctrine. It is everywhere affirmed. But only a few quotations are necessary here. Dr. Irving King declares that thinking has no other purpose than to solve the crises that arise within experience. "Science," says Schiller, "subordinates itself to the needs and ends of life, whether we regard its origin—practical necessity, or its criterion—practical utility." "The concrete truth *for us* will always be that way of thinking in which our various experiences most profitably combine" (James). Pragmatism is in complete sympathy with the scientific point of view, which holds according to James "that scientific theories are not copies of eternal truths, but are a conceptional short hand, true so far as they are useful, but no farther."

In pragmatism, then, we are confronted with a doctrine which in its statement at least seems open to the charge of subjectivism and pluralism; which clearly announces empiricism and utilitarianism, and which in these particulars far from being new is but a restatement—more careful and complete—of points of view centuries old. So far it is, in part at least, a revival of the anthropological categories of the Sophists; a restatement of the doctrine of Protagoras, who treated the problem of knowledge psycho-genetically, and as such it will be opposed to-day as it was opposed in antiquity. Schiller is certainly right in his opinion that "for a long time to come we must expect still to see the great principle of Protagoras maligned and perverted by those congenitally unfitted to appreciate it."

One of these congenital defectives of antiquity was Plato who felt that truth must be more than a merely

human affair, and who with one sweep of poetic imagination invented a world of pure ideas, changeless and abiding, and added thereunto the notion that all phenomenal realities were mere copies of these transcendent beings. Here we have absolutism and the "copy-theory" with a vengeance. Knowledge is the cognition of the eternal and necessary; experience is, indeed, a hindrance to such cognition. Thus did Plato attempt to escape from the dizzy whirl of "the instant field of the present." His was the first reply of the absolutists to the pragmatist, forceful but dogmatic.

It remained for a more systematic thinker, who had sympathy for both empiricism and absolutism, to set forth a more consistent theory of truth and reality. Aristotle was well fitted for his tasks. He was a scientist, who was also a philosopher. He recognized the value of empirical reality, but he clearly saw its hopeless limitations. In this great thinker we find the rare combination of the naturalist and the metaphysician. For him life was empirical and manifold and practical, but it was more; he saw the reality of the *νοσηματα θανάτου*, but he understood the necessity of *θεωρία* that attains to those highest truths that presuppose all knowledge. With wonderful clearness he recognized the value of empiricism and transcendentalism. He saw the worth of those practical activities that were to be measured by results, but he had too rich an experience to ignore the fact that in appearance we constantly strive after a universal element. So while not turning his back on experience, he asserted that the soul must possess a faculty of immediate knowledge, the postulates of all thinking. Real knowledge is gained through experience, but passes beyond the phenomenal world. In brief, Aristotle felt the force of the pragmatic fact that knowledge develops in experience, that as far as we know truth it is in evolution; but he also recognized the necessity

which thought imposed of thinking the relative in absolute terms—and then, impelled by the demands of thought to satisfy itself, he posited such an absolute, whose existence thus became as certain as that of thought itself, since to unthink it was, if consistency were prized, to end all thought.

Aristotle's logic was a central point in the thinking of the early Middle Ages, and hence arose the dispute concerning the meaning of logical relations and the conflict over universals. The nominalists of those days find their counterpart in the scientific pragmatists of to-day, who consider the categories of science to be valid only in so far as they serve as convenient means of manipulating the material with which they deal, but who hold the ultimate reality of these categories as a matter of entire indifference. The realists on the other hand are closely allied to the absolutists of the present generation, who are unwilling to admit that truth is merely truth in evolution, but who assert the ultimate and transcendent value of those conceptions which are designated as true. It was an adherent of realism, Anselm of Canterbury, who formulated the much abused and sadly misunderstood Ontological Argument to combat the nominalism of his opponents. We find this argument outlined in the *Proslogium*, and in brief it consists in the statement that the thought of God as the most perfect being involves the existence of God. The paraphrase of this argument by Guanilo, and later by Kant, made it appear ridiculous, but the real meaning of the truth here expressed was suspected neither by his mediæval nor his modern critic. The significance of the argument lies in the fact (clear to Aristotle) that the human mind must think something ultimate (we may call the ultimate God or what you will), and having thought the ultimate it must further affirm the existence of such an ultimate. There are only two paths open to the seeker after

reality. The reality is either mere subjectivity of sophism, or it is something that transcends subjectivity (not only my consciousness, but the consciousness of all finite individuals); further if we once allow ourselves to transcend this psychic immanence, we are forced to go on to that which is without limit or condition, to the highest species that embraces all below it (stated in logical terms), or to the ultimate cause lying behind all conditioning and temporal causes, to the metaphysical reality as distinguished from the mere occasional realities of the phenomenal world (stated in ontological terms). When all has been said and done the realist and the absolutist assert in opposition to the nominalist and empiricist that *something exists somewhere and somehow*. This is the one vital point of the entire problem, as vital to-day as when the question was formulated in Boethius's translation of Porphyry's Introduction. It is no new point of view, then, that pragmatism is upholding in this respect, but one which the human mind has found interesting and necessary since the classic period of Greek philosophy.

The metaphysics of pure experience is clearly affirmed in modern times by the great English empiricist Hume. All comes from sensation, and in the last analysis all ends with sensation, since no reality can be posited beyond, neither a material world nor a thinking substance. This radical empiricism contents itself with a purely positivistic point of view and is entirely in harmony with the pragmatism of to-day. Indeed the modern doctrine is a thoroughly legitimate development of the Humian standpoint, and completely in accord with a science of mind which finds states of consciousness all that psychology needs to do its work with.

Kant attempted to answer the subjectivism and empiricism of Hume. The result may be considered largely a failure as far the Critique of Pure Theoretical Reason

is concerned. Here we can know only phenomena. In his ethical treatises, however, Kant succeeds in transcending the individual and relative point of view and in getting beyond the world of change. Here he seems to assert the reality of those final ideals which remain mere ideals in his theoretical philosophy. They are demanded by faith and for faith are real. The same argument, however, holds good (though Kant did not apply it) in the theoretical sphere. Phenomena as such yield no satisfaction to thought, which demands the unconditioned, and hence must assert the unconditioned as a matter of intellectual faith, thus attaining a knowledge that is adequate to the Ideas of Reason.

This step the later German philosophy took in the idealistic optimism of Hegel and his predecessors. Thought overcomes all contradictions and imperfections and attains to the absolute which it demands from its own necessity.

Neo-Hegelianism states with wonderful clearness the fundamental motive of this conception, centuries old, in the proof of Professor Royce for the existence of God. It runs as follows:

"Reality as opposed to illusion, means simply an actual or possible content of experience, not in so far as this experience is supposed to be transient and fleeting, but in so far as it is conceived to be something inclusive and organized, the fulfillment of a system of ideas." But since we have no such experience as an actual fact in finite existence, we must argue to an Absolute Experience, to God. In other words, the fragmentary reality which human experience knows is unsatisfactory; but thought can not rest here. Since there is no perfect fulfillment of the ideal in the finite, there must be an infinite to realize such a fulfillment. Here we find again in a modern and much improved form, but still in essence the same as of old, the Ontological Argument of Anselm.

This, then, is the final point to which our discussion leads. Can we rest content in the "instant field of the present," or must we push out to the beyond? The pragmatist does not deny that such a transcendence must in part take place, but he would limit its scope to human experience. Once admit the principle of objective reference, however, and there is no stopping place. From the pure experience of the moment we push on to an experience complex with relations that yield no ultimate satisfaction to our thought. The phenomenal world of radical empiricism cannot fulfil our ideals and we are forced through sheer necessity (the same necessity which actuated Plato and Aristotle and Anselm and Kant and Hegel) to posit a world of eternal abiding reality beyond all hope of growth or shadow of change. We strive after the universal element in appearance. The "thought-crises" constantly arise and are never satisfied within individual experience. In contradistinction to this mode of thinking, pragmatism offers only a partial adjustment, "a relation of less fixed parts of experience to other relatively more fixed parts, and not a relation of experience to anything beyond it" (James). Such a point of view then must always remain unsatisfactory to that type of mind that holds up to itself the ideal of completeness; and there is no type of mind that does not, explicitly or implicitly, openly or covertly, possess to some degree such an ideal. This lack of the recognition of such a necessity is the one great defect of the pragmatic doctrine. On the other hand it expresses a great truth (one too often ignored by absolutism) that the detailed working out of our thoughts is in a field of practical, immanent reality in which all that is actual is either directly experienced or capable of statement in images derived from experience. Truth is not a copy of an external reality, but the expression of harmonies between elements of actual experience. So far

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the doctrine has great practical value. Such a truth, however, is transient and is not the ideal truth that our thinking demands. We can never, it is true, in human experience arrive at any other truth; hence the pragmatist asserts that this other truth, since it cannot be brought into relation with our lives, makes no difference and its existence or non-existence is a matter of no concern. That which, however, does make a difference is the attitude which we have toward the reality of such a transcendent truth. If we deny it, then we must admit that one truth is as good as another, since all are relative and transient, and no point of view, even if tested by ages of human thinking, can be nearer any abiding standard than another. In the infinity of possible experience any actual experience or set of experiences is infinitesimal. Indeed such a relative standard is condemned by the pragmatic criterion of satisfactoriness as impossible. *We must think the absolute, therefore the absolute exists.* And finally we have at the conclusion of our discussion this anomaly.

The pragmatist, if he is consistent, is obliged to posit for intellectual satisfaction an absolute, while on the other hand the absolutist, in asserting the reality of the transcendent, has no other warrant than that of the pragmatist, its practical necessity for thought.

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SOME NOTES ON THE IDEOGRAMS OF THE  
CHINESE AND THE CENTRAL AMER-  
ICAN CALENDARS.

ALEXANDER von Humboldt long since observed that:

"In Mexico from the seventh century until our era the days have been called 'tiger,' 'dog,' 'monkey,' 'hare,' or 'rabbit,' as throughout eastern Asia the years bear the same names among the Tibetans, the Tatar-Manchus, the Mongols, the Kalmüks, the Chinese, the Japanese, and among the nations of Tonkin and Cochinchina."

The learned scientist contrasted the day-names of the Mexican system with the year-nomenclature of the Asiatic races: sinologists have since demonstrated that the symbols now employed in China to designate the years were originally proper to the days alone, and that their use in connection with the "cycle of years" dates only from the first century of our era.

The invention of the system of time-measurement, current at the present day in China, Japan, Korea, Siam, Tibet, and the neighboring countries, is ascribed by Chinese tradition to Huâng-ti,\* B. C. 2636, though, according to Dr. Chalmers, the first known occurrence of the cyclical name of a day in Chinese literature is found in the *Shū Ching*—the "classic of history"—under the first year of

\*In transcribing Chinese sounds, the modern pronunciation of Peking is followed throughout these notes. Chinese characters inclosed in [ ] are antique forms.

the emperor T'ai-chia, B. C. 1752. "This is the only instance of the use of the cycle which occurs before 1121 B. C.," after which date it begins to be referred to by Chinese writers as a thing well established and known to all men.

These Chinese time-symbols, twenty-two in number, consist in two series of very ancient ideographic characters, obscure in origin and significance: one set of ten, known as "the celestial stems," and one of twelve, "the terrestrial branches." The commentary on the classical work called the "Record of Rites"—*Li Chi*—states that:

"Tá-não [a minister of the fabulous Huáng-tí mentioned above], having traced out the relations of the five elements and ascertained by divination the laws regulating the movements of the constellation of the Great Bear, invented the symbols *chià*, *yí*, etc., to name the days, calling this series 'stems'; and for naming the moons used the signs *tsù*, *ch'òu*, etc., which he styled 'branches.' By the combination of these 'branches' and 'stems' he completed the six decades of days."

Each of the twelve branches is under the tutelage of a particular animal, and the ten stems are assigned to various manifestations of the five elements—water, fire, wood, metal, and earth,—recognized by Chinese philosophy.

In ancient Mexico, a series of twenty symbols served to designate the days, and though, so far as we are aware, no division analogous to that of the Chinese stems and branches was made, we yet find that one-half of these Mexican characters bear the names of animals, and the remaining half the names of certain objects and influences in nature.

The following tables exhibit the arrangement of the two series of characters, Asiatic and American:

## I. THE CHINESE SERIES.

*A. The Ten Stems.*

NUMBER.	SYMBOLS		NAME.	ASSIGNMENT.	ELEMENTAL INFLUENCE.
	ANCIENT.	MODERN.			
1.	甲	甲	chià	fir tree	wood
2.	乙	乙	yi	bamboo	wood
3.	丙	丙	pìng	torch flame	fire
4.	丁	丁	ting	lamp flame	fire
5.	戊	戊	móu	hill	earth
6.	己	己	chì	plain	earth
7.	庚	庚	k'eng	weapon	metal
8.	辛	辛	hsin	cauldron	metal
9.	壬	壬	jên	wave	water
10.	癸	癸	kuèi	stream	water

*B. The Twelve Branches.*



NUMBER.	SYMBOLS		NAME.	ASSIGNMENT.
	ANCIENT.	MODERN.		
1.	子	子	tzù	rat
2.	丑	丑	ch'òu	ox
3.	寅	寅	yîn	tiger
4.	卯	卯	mào	hare
5.	辰	辰	ch'ên	dragon
6.	巳	巳	ssú	serpent
7.	午	午	wù	horse
8.	未	未	wéi	goat
9.	申	申	shen	monkey
10.	酉	酉	yù	cock
11.	戌	戌	hsü	dog
12.	亥	亥	hái	boar.

No. 10 is also written *yu*.<sup>1</sup> Several other forms of No. 12 are found.

<sup>1</sup> 酉[卯]

## II. THE MEXICAN SERIES.

NUMBER.	SYMBOL.	NAME	MEANING OF NAME.
1.		cipactli	dragon
2.		ehecatl	wind
3.		calli	house
4.		cuetzpalin	lizard
5.		coatl	serpent
6.		miquiztli	death
7.		mazatl	deer
8.		tochtli	rabbit
9.		atl	water
10.		itzquintli	dog
11.		ozomatli	monkey
12.		malinalli	grass
13.		acatl	cane
14.		ocelotl	tiger
15.		quauhtli	eagle
16.		cozcaquauhtli	vulture
17.		ollin	movement
18.		tecpatl	flint

19.		quiahuitl	rain
20.		xochitl	flower.

Von Humboldt considered the similarities between the two series so remarkable as to declare that:

"The six signs of the Tatarian zodiac (i. e. dragon, serpent, dog, hare, monkey, tiger) which are also found in the Mexican calendar, are sufficient to make it extremely probable that the nations of the two continents have drawn their astronomical ideas from a common source, and it is worthy of notice that the points of resemblance on which we insist are not derived from rude pictures or allegories susceptible of being interpreted in accordance with any hypothesis that it is desired to sustain."

The eminent Americanist, the late Dr. D. G. Brinton, takes issue with this pronouncement and finds cause to remark:

"Years ago, Alex. von Humboldt assigned it (the Central American calendar system) the first rank among the proofs that they (the American nations) had reached a certain degree of true civilization, indeed, so deeply did its intricacies impress him that he could not believe that it was wholly developed by tribes so uncultured in some other respects, and sought for its chief principles of origin among the civilizations of Asia. . . . A profounder study of the subject, rendered possible by more abundant documents, especially of a linguistic character, has shown that the hypothesis of the great naturalist is unnecessary, and indeed contrary to the evidence. The peculiarities which marked this calendar belong to itself alone, and differ completely from those on which the time-counts and astronomical measurements of the ancient nations of the Old World were based. It is strangely and absolutely independent in its origin and development."

So far Dr. Brinton.

Some of the peculiarities that marked the American system were: the beginning of the year at the winter solstice; the reckoning of time by a lunar as well as by a solar year, and in addition the employment of a ritual year of

260 days, or 20 periods of 13 days each; the division of the year into 72 periods of 5 days each, or 4 seasons of 18 weeks; the addition of intercalary days at the end of the year, by which the length of the solar year was fixed approximately at 365 days; the arrangement of the years in cycles by indictions, each year of the cycle bearing a compound designation, one element of which is the name of an animal.

In view of the fact that every one of these "peculiarities" exists also in the Asiatic calendar system, Dr. Brinton's assertion that the characteristics of the American calendar "belong to itself alone, and differ completely from those on which the time-counts and astronomical measurements of the ancient nations of the Old World were based," is susceptible of at least some degree of modification.

As a result of his "profounder study of the subject, rendered possible by more abundant documents, especially of a linguistic character," Dr. Brinton in his *Native Calendar System of Central America and Mexico*, undertook an exhaustive etymological analysis of the calendar names in the different central American languages, in the endeavor "to reach the symbolical significance of the calendar as a mythical record and method of divination." He designedly avoids any analysis of the written characters, believing that the aid thence to be derived is fallacious, and arrives at the conclusion that:

"Whatever other uses of an astronomical and time measuring character the calendar had, the best known and most general service which it rendered was for divining purposes. . . . The basic theory of the art of divination according to the calendar is nowhere stated. I propose to form a suggestion as to what this was, as appears to be indicated by the calendar itself, and to be supported by a number of collateral facts mentioned by early authors."

Dr. Brinton's suggestion is, that the 20 American calendar names constituted an esoteric résumé of the course

of human life and its vicissitudes. His linguistic analyses reveal that:

"Restoring the figurative terms to their literal meaning, we may conclude that the general and original symbolism of the day-names in all the tongues in which we have them, were as follows:

NUMBER.	SYMBOL.	HIERATIC SIGNIFICATION
1.	sword fish	birth, the beginning.
2.	wind	breath, life, the soul.
3.	darkness, the house	sleep, rest, repose.
4.	iguana	food, nourishment.
5.	snake	sexual life, reproduction.
6.	death	child-bearing, children.
7.	deer	hunting.
8.	rabbit, seed	agriculture.
9.	water, rain	illness or productiveness.
10.	dog	hardship and suffering, and success through them.
11.	monkey	difficulties surmounted.
12.	broom, teeth	loss, evanescence.
13.	reed	cold, drought, advancing years.
14.	tiger	learning, wisdom.
15.	eagle, bird	knowledge, skill.
16.	vulture, owl	old age, misfortunes.
17.	motion	debility, failing powers.
18.	flint knife	war, death.
19.	lightning	sickness, destruction.
20.	sun	the house of the soul."

Dr. Brinton continues:

"The examination of this sequence reveals that it was intended to cover the career of human life from the time of birth until death at an old age. The individual emerges from the womb of his mother and the parturient waters, as did the earth from the primeval ocean. He receives breath and with it life, which is supported by repose and food. The man reproduces his kind; the woman, at the risk of her life, brings the child into the world. The chase and tilling the ground are the leading occupations of peace; and he who holds firm through illness, suffering, and hardships, will gain the prizes of life. Having reached the acme of his career, a decline commences, losses



multiply, years increase, and though knowledge and wisdom are augmented, old age comes on apace, with failing powers, with vanquished struggles, with sickness and death; until at last, its course run, its task complete, the soul quits the worn-out body and soars to its natural haven and home, the abode of the sun. Such, it seems to me, without any straining, is the philosophical conception of life which was intended to be conveyed by the symbols of this strange old calendar. They may not have originated contemporaneously with it, certainly not if it was primarily deduced from astronomical observations, but quite probably if, instead of this, it was built on terrestrial relations and mythical concepts."

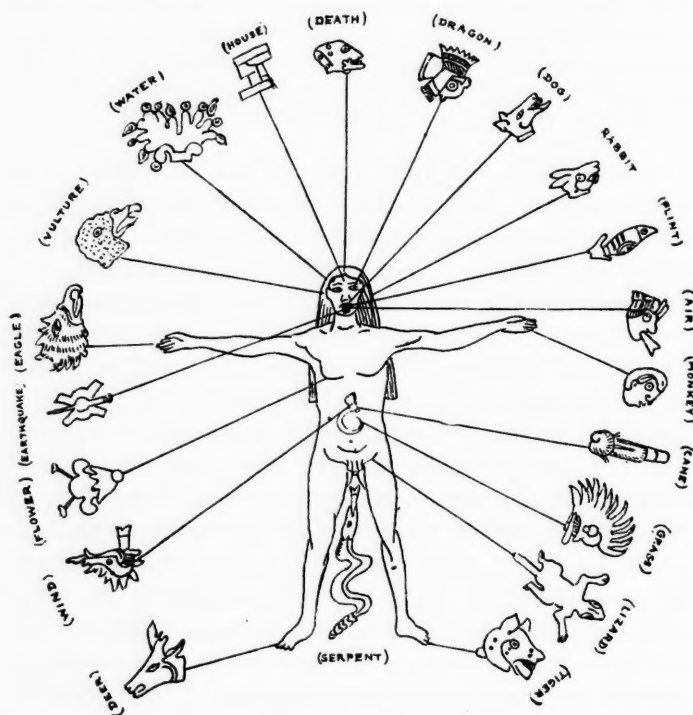
The question naturally presents itself: What can a similar analysis of the Asiatic day-names, based on "more abundant documents, especially of a linguistic character" than were available in Von Humboldt's time, reveal?

According to Dr. Brinton, the best known and most general service rendered by the American calendar was for divining purposes. It is unnecessary to advert to the connection between the cyclical ideograms and the art of divination in Asia. Every student of Chinese is familiar with the rôle played by the Ten Stems and the Twelve Branches in the astrologer's gramarye, and their never-ending reference to the points of the compass, the elements, the horary periods of the day, the various animals, the members of the body, and a thousand and one other alleged relations to the constituent parts of existence; and how upon their procession and interaction, in one order or another, are founded all the predictions of the diviner in regard to the secrets of nature and of being.

As a single instance. A figure in the Mexican manuscript catalogued as "Codex Vaticanus" exhibits the ascription of the day-emblems to the various parts of the human body. In Chinese almanacs of the present day may frequently be seen the pictures of the gods of the four seasons, their forms besprinkled with disks bearing the symbols of the Twelve Terrestrial Branches.

Dr. Carus very appositely suggests that similar presentments are common in mediæval European works on astrology and may be met with, not infrequently, in almanacs even of our own times.

Is it not curious to remark that in both these figures—



ASSIGNMENT OF DAY EMBLEMS TO THE PARTS OF THE HUMAN BODY.  
From Mexican Codex in Vatican Library.

American and Asiatic—the sign of the “tiger” is at the left foot; that the “hare” and the “cock” on the shoulders of the Chinese deity are paralleled by the “rabbit” and the “vulture” on the sides of the Mexican; and that the “lizard” in one and the “rat” in the other,—creatures which several

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circumstances tend to equate in cyclical nomenclature,—preside over the belly?

But to return. Are there any grounds for a belief that the Asiatic ideograms were intended "to cover the career of human life from the time of birth until death at an old age?" If this is so, the fact helps to strengthen very materially the evidence in favor of the derivation of the two calendars from a common source. The question is best answered by the characters themselves.



ZODIAC EMBLEMS ASSIGNED TO THE

CHINESE SUMMER GOD.

PARTS OF THE HUMAN BODY. From modern Chinese Almanac, substituting translations for the native characters of the twelve 'branches.'

In the Chinese field, we can not follow Dr. Brinton in ignoring the assistance to be derived from the written characters, for Chinese etymology is based not on phonetics but on a comparison and analysis of the oldest forms of the ideographic symbols.

Roughly speaking, each Chinese character is composed of two parts: one called a "radical" or "classifier," which is a conventional representation of the class of objects to which the character relates; and another, technically

known as the "phonetic complement," being a picture of some common object, usually totally unconnected in meaning, but the name of which has the same sound, with that of the object or idea denoted by the whole composite character. Consequently, from the very nature of Chinese writing, to quote Dr. Edkins,

"The sound of the 'phonetic' part of a character is an index to the sound of the words when the characters were first made.... Anciently, words like in their 'phonetic' symbols were like in sound. This is at once recognized by every one in simple cases.... We may proceed further than this, and say that when difficulties occur in discovering similarity in sound, it is in every case due to changes effected by time in the sounds of the words."

The modern Chinese sounds, in their myriad dialectic variations, can not afford a trustworthy basis for etymological deduction; the old phonetic symbols must be relied on for help in that direction.

#### ANALYSIS OF THE CHINESE IDEOGRAMS.

##### 1. *The First Stem, Chia.*<sup>2</sup>

Dr. Wells Williams, in his *Syllabic Dictionary of the Chinese Language*, defines this character as: "the plumule or scaly covering of a growing seed just bursting; cover of a sprout; a bud;" and says "the original character is described as composed of *mu*,<sup>3</sup> "wood," with a cap over it, representing the first movements of the sprout in spring." Dr. Chalmers prefers to regard *chia* as an original ideogram, and makes no analysis of its component parts. His definition is: "a protecting cover; helmet." It may be remarked, *en passant*, that the Chinese, Aztec, and Maya day-symbols correspond in so far at least that they are all non-composite, or what sinologists would term "primitive" characters. The Imperial K'ang-hsi Lexicon tells us that "grass and trees beginning to grow" are called

<sup>2</sup> 甲[𠂔]

<sup>3</sup> 木[𣎵]

by this name *chia*, and among ancient forms of the character listed by that authority occurs one,<sup>4</sup> in which *jen*,<sup>5</sup> the ideogram for "man," is placed on top of the sign *chia*<sup>6</sup> referring directly, as it would seem, to the opening period of human life. Surmounted by *jih*,<sup>7</sup> "the sun," similarly the character *tsao*,<sup>8</sup> has the meaning "early." Surrounded by *men*,<sup>9</sup> "the two leaves of a folding door," its significance is the same,—“the door just open to admit the rays of the rising sun.”<sup>10</sup> There is also a close resemblance between the old form of this stem *chia*<sup>11</sup> and that of *ch'üan*,<sup>12</sup> "a fountain," "a spring." The upper part of both characters is like *mi*<sup>13</sup> (old sound *mik*), "a protecting cover," which enters frequently into the composition of Chinese symbols (Cf. the Maya *mac*<sup>14</sup> of supposedly similar name). Etymologically connected may be: *chia*,<sup>15</sup> "to hide away," "put in the bosom," "cherish"; *chia*,<sup>16</sup> "a metal undershirt." The general idea conveyed by this first stem and its derivatives is something hidden awakening into life,—the commencement of plant life in particular.

## 2. The Second Stem, *Yi*.<sup>17</sup>

This is defined as "a curling sprout or bud just coming out of the darkness and seclusion of winter." Chalmers says "the bursting of vegetation...this (ideogram) denotes effort in *ch'ien*,<sup>18</sup> 'dry,' 'strong,' 'advancing'; *luan*,<sup>19</sup> 'to unravel,' 'confusion,' and *yu*,<sup>20</sup> 'excess.'" In the K'ang-hsi Lexicon occur two very similar characters, now obsolete, *ya*,<sup>21</sup> defined as *nan ch'u chih mao*, "the appearance of difficulty in coming forth," and *yin*,<sup>22</sup> described as an old form of *yin*,<sup>23</sup> "to conceal." These and the stem symbol depict very plainly: (i) the motionless plant hidden in the

4 令	5 人	6 甲	7 日	8 早[早]	9 門	10 開開
11 𠂔	12 泉[泉]	13 一[一]	14 𠂔	15 挾	16 輪	17 乙[乙]
18 乾	19 亂	20 尤	21 乙	22 乙	23 隱	

ground,<sup>24</sup> and the different forms it takes when sprouting under (ii) favorable<sup>25</sup> and (iii) unfavorable<sup>26</sup> circumstances. The stem name is etymologically connected with *yi*,<sup>27</sup> "one," "the first." The K'ang-hsi lists an old form of *jih*, "the sun," which consists in a circle enclosing this second stem,<sup>28</sup> i. e., the one circle, the great disk. Other connections, real or imaginary, are: "to let loose"<sup>29</sup>; "to overflow";<sup>30</sup> "to rush on," as to battle;<sup>31</sup> "to unfold," "to fly"<sup>32</sup>; "standing ready"<sup>33</sup>; "coming suddenly"<sup>34</sup>; "full"<sup>35</sup>; "exuberant"<sup>36</sup>; "to increase"<sup>37</sup>; "to pour out"<sup>38</sup>; "to be ready to overflow"<sup>39</sup>;—all now pronounced *yi*. The ideas conveyed by the second stem, then, are (i) the uncurling of a sprout and coming out from darkness, and (ii) oneness, unity, the consciousness of the ego.

### 3. The Third Stem, Ping.<sup>40</sup>

One authority says: "Composed of *yi*,<sup>41</sup> 'one,' *ju*,<sup>42</sup> 'to enter,' and *chiung*,<sup>43</sup> 'a receptacle or door'; *yi*<sup>41</sup> representing the *yang*<sup>44</sup> principle." In other words, this is the sun entering the door, the dawn of reason in the individual. *Yi*, "one," is here used as the equivalent of the old sun symbol alluded to under the last stem. In modern Chinese and Japanese, the sun is styled *t'ai yang*,<sup>45</sup> *taiyo*, "the great positive principle." The dictionary *Shuo-wen T'ung-hsün Ting-sheng* arrives at the same conclusion by slightly different analysis. This character, it says, is the original of *ping*,<sup>46</sup> "fiery," "bright," "conflagration," and *tsai*,<sup>47</sup> "fires under a cover," is an old form of the same. The top line is the same as *shang*,<sup>48</sup> "to ascend," and the rest is "fire under a roof" contracted. The concept of "brightness" and "flame" runs through the cognate characters,

24 𠂔	25 乙	26 乚	27 一	28 囧	29 遠	30 洪	
31 𢇛	32 擇	33 𠂔	34 𠂔	35 𠂔	36 翼	37 益	38 挹
39 滄	40 丙	41 一	42 𠂔	43 𠂔	44 陽		
	45 太陽	46 炳	47 𠂔	48 上			

and the stem symbol itself is used in Canton to write the local word *ping*, "to burn." Surmounted by the "sun" ideogram, the stem means "bright and glorious like the sun," and has the same name, *ping*.<sup>49</sup> Infancy is now past and the child is advancing into the brightness of youth,—such seems to be the meaning conveyed by the symbol. Possible connections are: *ping*,<sup>50</sup> "bright," "luminous"; *ping*,<sup>51</sup> "to have power"; *ping*,<sup>52</sup> "power"; *ping*,<sup>53</sup> "to confide in"; *p'ing*,<sup>54</sup> "undisturbed."

#### 4. The Fourth Stem, *Ting*.<sup>55</sup>

This is the figure of a nail or peg, anything firmly fixed or settled. Dr. Williams says it denotes "that things are perfected... robust and full grown person... to sustain, to bear." Another old form of the character is composed of *jen*,<sup>56</sup> "man," over *chüeh*,<sup>57</sup> "hook," the latter said to stand here for *hsin*,<sup>58</sup> "the heart;" the man's heart is established, full of determination and courage. With the addition of the sign for "weapon," indicating martial prowess, the character *ch'eng*<sup>59</sup> has the meaning "complete," "perfected." A character<sup>60</sup> occurring on a bell of the Shang dynasty, cited by Edkins, and identified with this *ting* stem, bears some resemblance to the *ch'eng* symbol. On a tripod of Fu-ting is found a quite dissimilar form,<sup>61</sup> which may represent the head of a nail; a third old form,<sup>62</sup> is a very fair picture of the nail in full. This last ideogram is identified by some with *liang*,<sup>63</sup> "honest," "upright." In Peking, a young man, sixteen to eighteen years of age, is styled *ting-nien*,<sup>64</sup> "arrived at manhood," and this is exactly the idea appropriate to the position occupied by the *ting* stem in the sequence. Related words are numerous: *ting*,<sup>65</sup> "a nail," "a spike"; *ting*,<sup>66</sup> "firmly set-

49 𠂔 50 炳 51 乘 52 柄 53 凭 54 平 55 丁(个)

56 人(刀) 57 丿(匕) 58 心(心) 59 成(咸) 60 𠂔 61 𠂔 62 𠂔

63 良 64 丁年 65 釘 66 豐



tled"; *ting*,<sup>67</sup> "peak," "summit"; *ting*,<sup>68</sup> "secure," "fixed," "steady"; *ting*,<sup>69</sup> "ballast"; *ting*,<sup>70</sup> "honest," "trustworthy"; *t'ing*,<sup>71</sup> "decided," "resolute"; *t'ing*,<sup>72</sup> "baton of authority"; *t'ing*,<sup>73</sup> "good," "complete," "full," "whatever is the purpose of life."

### 5. The Fifth Stem, *Wu* or *Mou*.<sup>74</sup>

Supposed to be a form of *mou*,<sup>75</sup> "an ax or halberd," but probably used phonetically for *mou*,<sup>76</sup> "luxuriant," "ripe grain," "when all things are flourishing," with which both Edkins and Williams connect it. The characters *ch'eng*,<sup>77</sup> and *hsien*,<sup>78</sup> written with the same phonetic element, mean "complete," "perfect," "all." Connections with *mou*,<sup>79</sup> "luxuriant"; *mou*,<sup>80</sup> "force of purpose," "to exert one's mind"; *mou*,<sup>81</sup> "male," "virility"; *mou*,<sup>82</sup> "to deliberate," "to ponder"; *wu*,<sup>83</sup> "to bend the mind to a subject"; *wu*,<sup>84</sup> "to gallop furiously"; *mou*,<sup>85</sup> "to do business," "carry on commerce";—are more or less probable. The leading idea is activity, the business of life.

### 6. The Sixth Stem, *Chi*.<sup>86</sup>

The native lexicographers do not explain the form of this symbol, but all are agreed as to its signification: "one's self." It resembles ideograms employed to depict "breath" and "air." Williams says: "This character is connected with the center of a thing, as it is considered to be altered from *chung*,<sup>87</sup> "the middle," and because it is the sixth of the ten stems." Of similar meaning are *chi*,<sup>88</sup> "to exhaust a subject"; *chi*,<sup>89</sup> "to stand up." The man is now "himself," has come to his own, having accomplished the purpose of life, and the gradual descent to the grave begins.

67 頂	68 定	69 訂	70 停	71 挺	72 挺	73 壬
74 戊戌	75 𠂔	76 𠂔	77 𠂔	78 𠂔	79 𠂔	80 𠂔
81 杜	82 謀	83 務	84 鷲	85 賀	86 己(己)	87 中(中)
		88 紀	89 起			

7. The Seventh Stem, Keng.<sup>90</sup>

"The original form represents the hands receiving a thing, as at autumn when all things are full...to change, to alter, age; to bestow reward." In Chalmers' opinion it is composed of *kung*,<sup>91</sup> "to lift up the hands together," and *kan*,<sup>92</sup> "to violate"; the latter element being phonetic. *Kan* itself is composed of *ju*,<sup>93</sup> "to enter," inverted, and *yi*,<sup>94</sup> "one." His definition is: "joining on to, as the border or foot of a garment." The main idea seems to be either (i) change, alteration; or (ii) rewards of labor; *cozca-quauhltli* and *ollin* are the American analogs. Connected may be: *keng*,<sup>95</sup> "to change"; *keng*,<sup>96</sup> "a limit," "an extreme point"; *keng*,<sup>97</sup> "a path leading to a sepulchre"; *keng*,<sup>98</sup> "to thrum the threads of a lute rapidly, so as to endanger breaking them."

8. The Eighth Stem, Hsin.<sup>99</sup>

From *yi*,<sup>100</sup> "one," and *ch'ien*,<sup>101</sup> "error," explained as depicting "the arms of a man holding up a thing, referring to the sorrow one feels at winter coming." *Ch'ien* is from *kan*,<sup>102</sup> "to violate," under an old form of *shang*,<sup>103</sup> "superiors." The ordinary meaning of *hsin* is "a bitter, sharp, pungent taste"; whence, by metonymy, because the peppery taste makes the tears flow, "toilsome," "suffering," "grievous," "sad," "the melancholy feeling in autumn when vegetation turns sere." The Shuo-wen says it is composed of *ch'ien*,<sup>104</sup> "to offend superiors," plus an extra stroke,<sup>105</sup> showing the enormity of the crime. The meaning of sadness and failing powers is appropriate to the position occupied by the character in the stem series.

90 庚[庚]	91 甲[甲]	92 干[干]	93 入[入]	94 一	95 更	
96 瓦	97 壘	98 施	99 幸[幸]	100 一	101 [幸]	102 [幸]
		103 [二]	104 [幸]	105 [一]		

9. The Ninth Stem, Jen.<sup>106</sup>

According to Dr. Williams, "defined as a man standing on the earth, the earth denoting the business of life." Others say it represents the embryo in the womb. It resembles *t'ing*<sup>107</sup> in form, and as a phonetic element is often interchanged with *jen*,<sup>108</sup> "sincere." Other meanings are: "running water," "great," "full," "to flatter." Edkins' account is: "Plants growing out of the ground, with the sense *t'ing*, 'grow upward.' This stem is very similar in form to *t'ing* and indeed seems to be confounded with that symbol by some of the authorities." Chalmers' definition of the *jen* stem is "to sustain, to bear. . . probably a derivative of *kung*,<sup>109</sup> "work," like *chü*,<sup>110</sup> *wu*,<sup>111</sup> *ch'en*,<sup>112</sup> and *ya*,<sup>113</sup> and the middle line denotes the person doing the work intended." The character, as now written, consists in the sign for "scholar" with an additional stroke on top. Concerning the *t'ing* ideogram, Chalmers says that the old form<sup>114</sup> shows a man standing on the soil, meaning "to stand up," "go." It is probable that the *jen* sign is used phonetically for a character, now read *jen*,<sup>115</sup> meaning "sincere," "sure," "trusty," "rely upon," "a trust," "an office," "to undertake," "be responsible for," "the incumbent," "acting official." Other words with the sound *jen* have significations suitable to the position of this emblem among the stems; e. g., *jen*,<sup>116</sup> "to dwell on with satisfaction," as a well-spent life, we may opine; *jen*,<sup>117</sup> "fortitude," "endurance"; *jen*,<sup>118</sup> "grain which is fully ripe"; *jen*,<sup>119</sup> "the yellowish color of an old sword." The significance of this symbol lies in its reference to the completion of the affairs of life and to pleasurable retrospection.

106 壬[王]	107 壬	108 任	109 工	
110 巨	111 巫	112 臣	113 亞	114 [王]
	115 任			
	116 巷	117 忍	118 稔	119 狃

10. *The Tenth Stem, Kuei.*<sup>120</sup>

The old form in the shape of a cross is, according to the native etymologists, the same as the modern *kuei*,<sup>121</sup> "arms," "tridents," "arrows," etc. The modern form,<sup>122</sup> from the Li<sup>123</sup> writing, is also old, and is said to be a combination of *po*,<sup>124</sup> "to stop," "to hinder," (from *chih*, "to step," in two positions), with *shih*,<sup>125</sup> "an arrow," i. e., to send home an arrow. The dictionary Shuo-wen, commenting on the old "cross" form, points out that it represents "water flowing from the four quarters and entering the ground." Williams states that "the original form is like two sticks laid across to represent water flowing into the ground in all directions." Edkins, following the Liu-shu Cheng-wei, thinks that the two pieces of wood placed crosswise "formed no other than an ancient implement used in leveling." This was called *kuei*,<sup>126</sup> and used by builders in reducing land to a level. In the variety written with the arrow sign, Edkins detects *pei*,<sup>127</sup> "north," and *shih*,<sup>128</sup> "arrow." The north belongs to winter, and *kuei* is applied to both. "Both earth and water," on the authority of the Shuo-wen, "then become smooth and flat and can be easily measured." Streams flowing together are now *kuei*,<sup>129</sup>—a similar sound but differently ideographed. A place where four roads meet is also *kuei*.<sup>130</sup> All the authorities identify the lower part of the *kuei* stem with the "arrow" sign, but the forms of the two are not quite the same. (See Wu-yin Chih-yün and Chih-yün in K'ang-hsi, s. v. *kuei*.<sup>131</sup>) The etymological connections are very interesting and suggestive. That which most naturally presents itself is,—as suggested by the Cheng-yün,—"to return," *kuei*,<sup>132</sup> "to go to," "to send back," "to revert to the original place,"

120 癸	121 戔	122 癸	123 隸	124 隸
125 𠂔	126 規	127 北	128 矢	129 滙
		131 𠂔	132 歸	130 達

used in the phrase *kuei wu*,<sup>133</sup> "to revert to nothingness," "to die." Other relations are: *kuei*,<sup>134</sup> "strength all gone out"; *kuei*,<sup>135</sup> "water all dried up," as in a fountain or well; *kuei*,<sup>136</sup> "to change," "to alter"; *kuei*,<sup>137</sup> "the day, but especially the shadow," as in *fei kuei*,<sup>138</sup> "time flies." The tortoise, *kuei*,<sup>139</sup> emblem of longevity, and the juniper tree *kuei*,<sup>140</sup> whose durable wood is prized for making coffins, as also *kuei*,<sup>141</sup> "the spirit of a dead man," are all called by this same name *kuei*; and it is curious to note that the "disk of the sun" *kuei*,<sup>142</sup> and the "sunflower" *kuei*,<sup>143</sup> are similarly designated, in connection with the fact that the last day-name in the American calendars is "sun," the home of the soul, otherwise styled *xochitl*, i. e., "flower."

Summarizing the results attained by these analyses of the "Ten Stems," we obtain a sequence somewhat as follows:

1. *Chià*; the sprout is still hidden in the ground, but begins to feel the impulse of life and shoots upward;
2. *Yi*; it has reached the surface and uncurls;
3. *Ping*; the sun reaches it; and beneath his influence
4. *Ting*; it stands upright; and gaining vigor,
5. *Móu*; spreads forth luxuriantly; until, as a tree,
6. *Chì*; it has reached its full growth; and
7. *Kēng*; its fruit is gathered;
8. *Hsin*; decay sets in; the leaves fall;
9. *Jên*; the seed is again hidden in the ground; and
10. *Kuèi*; at the close of the year, there is a return to the original darkness, and the processes of nature enter on a new cycle of life.

As the "ten stems" have reference to the operations of nature in the procession of the seasons, so it can be shown that the "twelve branches" relate to man's affairs in particular.

133 歸無	134 効	135 曆	136 桅
137 罍	138 飛罍	139 龜	140 椿
	142 規	143 葵	141 鬼

A. The First Branch, Tzu.<sup>144</sup>

This is explained as the "figure of a baby strapped on its mother's back. The legs are swathed together in their wrappings, hence represented by a single stroke. An old form<sup>145</sup> is recorded, where the three vertical lines on top show the hair. The character presents no difficulty; it is in everyday use, with the meaning "child," "son."

B. The Second Branch, Ch'ou.<sup>146</sup>

The original form resembles a hand holding things. Others say: "it is like *shih*,<sup>147</sup> 'ten,' inside of *erh*,<sup>148</sup> 'two,' because the twelfth month is called *ch'ou yüeh*."<sup>149</sup> Further old forms are *ch'ou*<sup>150</sup> and *ch'ou*.<sup>151</sup> The latter may be compared with the Maya sign,<sup>152</sup> supposed by some to be a "hand." The radical idea is holding, grasping, guiding; referring perhaps to the care of the parents for their offspring. Compare the ideograms of "grasping a son"<sup>153</sup> and "grasping a daughter,"<sup>154</sup> both synonymous with *hao*,<sup>155</sup> "to love." Of like sound and meaning with the branch character are *ch'ou*,<sup>156</sup> all expressive of "to hold"; *chou*,<sup>157</sup> "to environ," "to provide for"; *chou*,<sup>158</sup> "to shade," "to conceal"; *ch'ou*,<sup>159</sup> "to hold, as the earth does"; *ch'ou*,<sup>160</sup> "to conceal by holding in the elbow"; *ch'ou*,<sup>161</sup> "to take out with the end," "to lift"; *ch'ou*,<sup>162</sup> "to arrange details."

C. The Third Branch, Yin.<sup>163</sup>

Williams, citing some metaphysical abstractions of the Shuo-wen, says: "From *mien*,<sup>164</sup> 'a covering,' which is likened to the kneepad that prevents the humors from ascending the body. These humors are depicted by *chin*,<sup>165</sup>

144 子[子]	145 [𠂔]	146 丑[𠂔]	147 十	148 二	149 丑月	
150 [𠂔]	151 [𠂔]	152 [𠂔]	153 𠂔	154 政	155 好	156 紐紐
157 周	158 𠂔	159 由	160 肘	161 抽	162 袖	
	163 寅[寅]	164 𠂔[𠂔]	165 𠂔			

'a mortar,' as coming out of the ground, and including the stimulus of nature in spring which the frost hinders . . . 'to reverence,' 'to respect,' 'respectful,' 'a fellow officer,' 'a colleague,' 'vigorous,' 'strong.' This is not over-intelligible to the Occidental mind. In the oldest forms, the character is composed of a central part, said to be a modification of *jen*,<sup>166</sup> the "man" sign; *chü*,<sup>167</sup> "the hands brought together," "to clasp the hands"; and *mien*,<sup>168</sup> "a house," "a covering." With this compare the character *hsüo*,<sup>169</sup> "to educate," where may be seen the "child," *tsu*,<sup>170</sup> under a "shelter," *mien*;<sup>171</sup> above, the signs of "imitation," *hsiao*,<sup>172</sup> and of "guiding hands," *chü*;<sup>173</sup> intimating that, in the process of education, the protected child is guided and taught to imitate. Suggested connections: *yin*,<sup>174</sup> "to take an interest in"; *yin*,<sup>175</sup> "to shelter"; *yin*,<sup>176</sup> "to regulate," "to sustain"; *yin*,<sup>177</sup> "to lead on," "to point out," "bring forward"; *yin*,<sup>178</sup> "careful," "anxious"; *yin*,<sup>179</sup> "to move forward," "to journey"; *yin*,<sup>180</sup> "to nourish"; *yin*,<sup>181</sup> "covered," "in private life," "not in office." To this branch may be allotted the meaning of early studies, the child being instructed in the respect due to its parents. With this *ch'ou* symbol compare *fu*,<sup>182</sup> "father," where a hand grasping the rod of authority is pictured.

#### D. The Fourth Branch, Mao.<sup>183</sup>

The old form is like an open door, or rather the two leaves of a double door, "analogous to the springing up of vegetation in March." It is defined by *mao*,<sup>184</sup> "a cover," as the earth is then covered. This last symbol, *mao*, "a cover," has, among other meanings, that of "rushing heedlessly," "venturing," "going forward," and the branch-sign itself is used in divers binomial expressions with a

166 [天]	167 [扌]	168 [人]	169 學	170 [弓]	171 [門]
172 [𠂔]	173 [𠂔]	174 𠂔	175 陰	176 殷	177 引
178 𠂔	179 𠂔	180 𠂔	181 陰	182 父	183 𠂔
184 𠂔					



like signification, as "calling the roll," *tien mao*,<sup>185</sup> "answering a summons," *chou mao*.<sup>186</sup> The symbolism may be the venturesomeness of youth or the responding to the roll-call of duty when the period of instruction, typified by the preceding branch, is over.

*E. The Fifth Branch, Ch'en.*<sup>187</sup>

The ancient form is supposed to depict "sprouting plants transformed by heaven... 'to excite,' 'to occasion,' 'to move,' 'to influence.'" Such is Dr. Williams' definition. The native lexicon *Shuo-wen T'ung-hsün Ting-sheng* says that the old form is composed of the sign for a human being, and another, meaning something concealed, the whole having the meaning "pregnant."

*F. The Sixth Branch, Ssu.*<sup>188</sup>

This is the picture of a serpent, the animal emblematic of the sixth branch. It refers to the fourth moon, says one authority, when all nature is in vigor. The ideas conveyed by this symbol and that of the last branch correspond with those associated in Mexico with the skull and serpent, *miquiztli* and *coatl*, motherhood and fatherhood.

*G. The Seventh Branch, Wu.*<sup>189</sup>

The figure of a pestle, defined as expressing "the resistance which the earthy vapors of the fifth moon, hence called *wu yüeh*,<sup>190</sup> oppose to the skyey influences, thus covering the earth with fog." This sign is used with *wu*,<sup>191</sup> "to oppose," "to stand up," "to resist," "cross," "transverse." Chalmers points out that the seventh branch has the sense of crossing. It denotes the sun at noon, and hence the meridian of life. Its form is very similar to that

185 點卯

186 畫卯

187 辰圖

188 巳圖

189 午[中]

190 午月

191 迂

of the "stem" called *ting*,<sup>192</sup> and both may possibly have been symbols of reproduction.

*H. The Eighth Branch, Wei.*<sup>193</sup>

This is *mu*,<sup>194</sup> the "tree" symbol, with an extra line at the top, showing the abundance of foliage and the tree's full vigor in the sixth month. So say the lexicologists. The character is a common adverb of negation and doubt, "not yet." Although the midday has passed, the set time has not yet expired, *meh ssu meh le li*, as the Shanghai vernacular has it. This is the counterpart of *mou*<sup>195</sup> among the "stems."

*I. The Ninth Branch, Shen.*<sup>196</sup>

"Formed," says Willims, "of *chiu*,<sup>197</sup> 'a mortar,' and *kun*,<sup>198</sup> 'to join.' Others say the character is intended to represent the backbone... 'to extend,' 'to stretch,' 'reiterate,' 'prolong,' 'increase.'" The Shuo-wen says that the element which Williams makes out as *chiu*,<sup>197</sup> "a mortar," is *chü*,<sup>199</sup> "the two hands holding or grasping" (found in the third branch *yin*<sup>200</sup>), and that the stroke *kun*<sup>198</sup> pictures the body; the whole representing a man placing his hands to his sides and stretching himself. The character is emblematic of weariness and declining years, as in the eighth stem.

*J. The Tenth Branch, Yu.*<sup>201</sup>

One old form is a figure of a vessel used for distilling, "referring to the closing up of nature in the eighth moon, when crops are ripe... 'ripe,' 'finished,' 'mature,' 'as ripe millet fit for making spirits,' 'the ripeness of crops.'" This branch is assigned to the west, and bears a great likeness to some old forms of the symbol *hsi*,<sup>202</sup> "west," employed

192 丁[𠂇] 193 未[耑] 194 木[耑] 195 戊 196 申[申] 197 巳  
198 丨 199 𠂇 200 寅 201 酉[酉] 202 西

in writing the name of that cardinal point. *Hsi*, however, is said to picture "a bird sitting on its nest at sunset." Another old form<sup>203</sup> of this branch shows "a door barred," the antonym of the fourth branch,<sup>204</sup> which is "an open door." This typifies the closing, as the other does the opening, of life. Some connected words: *yu*,<sup>205</sup> "an old building whose timbers are decayed;" *yu*,<sup>206</sup> "dark," "obscure," "hidden from view," "shades or spirits who are in obscure places"; *yu*,<sup>207</sup> "grieved," "mournful"; *yu*,<sup>208</sup> "to float," "to travel to," "go away."

#### K. The Eleventh Branch, *Hsü*.<sup>209</sup>

This is *wu*, the fifth stem, with the addition of a short horizontal line, perhaps denoting the wound made by the weapon, or the movement in throwing the spear. Compare other symbols of motion, such as *mou*,<sup>210</sup> "the long-handled ax attached to a chariot," where the dictionary makers tell us that the waved line is indicative of the revolutions of the instrument; also *ch'ou*,<sup>211</sup> "to plow," showing the furrows and the "boustrophedon" movement; also *sheng*,<sup>212</sup> "to ascend," where the three short lines are said to show the steps of the sun on his upward path. The meaning of this branch is "hurt," "pitiable," correlative with the *hsin* stem, and referring to nature fading in the ninth moon and the closing of human life. Others say it is symbolical of "fullness, for things start in *wu* (the fifth branch) and get ripe in *hsü* (this eleventh branch) when the sun's heat declines."

#### L. The Twelfth Branch, *Hai*.<sup>213</sup>

The Shuo-wen says this is just another form of *shih*,<sup>214</sup> "a pig." Williams thinks it is made of *jen*,<sup>215</sup> "man" above,

203 𠂔	204 𠂔	205 𠂔	206 幽	207 憂	208 幽
209 戌	210 亥	211 𠂔	212 𠂔	213 亥	214 豕
		215 人			

and *nü*,<sup>216</sup> "woman" below. None of the authorities gives any satisfactory explanation of the character, and its old forms are quite numerous and dissimilar. The sound *chu*<sup>217</sup> means "to drive out," and *sui*<sup>218</sup> means "to scatter"; both of these are written by the "pig" symbol, so that it is probable, or, on Edkins' theory of the phonetic complements, almost certain that the old sound of these words was *shih*, or whatever the old-time name of the hog may have been. It is not improbable then, that this *hai* branch



may have had the meaning "drive away," "scatter," although, etymologically, it seems rather to be connected with words meaning "young child." (Cf. also *chu*,<sup>219</sup> *chu*.<sup>220</sup>) One old form<sup>221</sup> contains, apparently, the elements *jen*,<sup>222</sup> "man," and *yin*,<sup>223</sup> "to conceal," which might lead to the supposition that the character has reference to the tomb. This explanation is offered with hesitation, as it rests on no native authority.

216 女[𡚦]

217 逐

218 遂

219 豕

220 猪

221 𡚦

222 [人]

223 [冫]

The Twelve Branches are assigned to the cardinal points, beginning at the north, the region of winter and darkness; passing around through the east (spring), the south (summer), to the west (autumn), and thus back to the north again, as shown in the foregoing diagram.

In this arrangement the symbol of the opening door in the east and that of the closing door in the west are immediately noticed. The position of the two characters *yin*<sup>224</sup> and *shen*,<sup>225</sup> each containing the "hand" sign, also arrests the attention. This placement of contrasting emblems was evidently premeditated and offers a clew to the interpretation to be given to some of the ideograms whose significance is more or less obscure.

The child, *tzu*,<sup>226</sup> in the north, is set opposite the man in the prime of life, *wu*,<sup>227</sup> in the south. (Cf. the form and meaning of the stem *ting*<sup>228</sup>). The restraining hand, *ch'ou*,<sup>229</sup> stands over against the tree in its unrestricted exuberance of foliage, *wei*.<sup>230</sup> *Yin*,<sup>231</sup> where may be seen the child, *jen*,<sup>232</sup> emerging from the home, *mien*,<sup>234</sup> as yet sustained by the parental hands, *chü*,<sup>235</sup> and exulting in his new-found strength, is opposed to *shen*,<sup>236</sup> the man stretching himself in world-weary languor. Then come the opening and the closing doors; and next *ch'en*,<sup>237</sup> a symbol of the beginning of life, in opposition to *hsü*,<sup>238</sup> indicative of approaching dissolution. While to complete the series there is the antonymy of *ssu*,<sup>239</sup> "the serpent," a genetic sign, as against *hai*,<sup>240</sup> "the man concealed," alluding, as may be supposed, to the last home and resting-place of the soul.

Several circumstances tend to show that the eleventh and twelfth branches originally occupied a different place in the series, and that the sequence began with what is now

224 寅[寅]	225 申[申]	226 巳[巳]	227 午[午]	228 未[未]	229 辰[辰]
230 卯[卯]	231 酉[酉]	232 戌[戌]	233 子[子]	234 丑[丑]	235 寅[寅]
	236 卯[卯]	237 辰[辰]	238 巳[巳]	239 午[午]	240 未[未]

the third symbol; but this fact does not materially alter the significance of the series, since it is to be regarded as continuous and never-ending, a delineation of the ceaseless interaction of the positive and negative principles in the great scheme of being.

#### RECAPITULATION.

Having thus predicated a community of intention on the part of the framers of these cycle-names in regard to the lesson to be conveyed by the sequence of symbols, the attempt may perhaps be made to compare the individual characters of the two continents yet more closely.

The first stem, *chia*, represents the seed beginning to sprout under the influence of the sun's rays in spring. One of the derivations proposed for the Maya, Tzental, and Quiché-Cakchiquel name of the first day, *imix*, *imox*, connects it with *mex* or *mix*, "the beard," and metaphorically "the sun's rays."

The second stem, *yi*, shows a young plant coming out from darkness into the air and light of day, and this ideogram also conveys the idea of individuality, self-consciousness, the dawn of reason. In the Zapotec calendar the second day was *ni*, "to grow," "increase," "gain life," or *laa*, "warmth," "heat," "reason," "intelligence." In the Maya, Tzental, Nahuatl, Quiché-Cakchiquel, (and also sometimes in the Zapotec), the second day is *ik*, *igh*, *ik*, *gui*, *ehcatl*, all meaning "air," "life," "soul."

The third stem *ping* pictures the sun entering a house or covering. The third Mexican day is *calli*, "house."

The fourth stem, *ting*, is an emblem of virility and reproduction. *Cuetzpalin*, the lizard, is the fourth Mexican day, and this creature was the ruler of the womb and loins. In Tzental the fourth day was *ghanan*, a name referring to the god of plants and abundance.

*Chi*, the sixth stem, is, as has been seen, considered by

some Chinese authorities as a variant of *chung*, "the middle," "the center." The Zapotec *lana*, meaning "separate," "apart," "the middle," and the Tzentel *tox*, "what is separated," are applied to the sixth day in the American series.

The seventh stem, *keng*, contains a "hand" sign, as does also the symbol for *manik*, the seventh Maya day.

*Jen*, the ninth stem, is referred to the pregnant womb and is assigned to the element water. In Maya and Tzentel, *muluc*, *molo*, names of the ninth day, mean "to pile up," "to heap up," while the Zapotec term *niza* and the Nahuatl *atl* both mean "water," and are employed as names of the ninth day in those calendars.

It was seen above that the interpretation ordinarily given to the seventh branch of the Chinese series was "pestle." In the Meztitlan dialect of the Nahuatl, the sixteenth day was known as *temetlatl*, which also signifies "pestle."

It will be noticed that the Ten Stems of the Chinese series are distributed by pairs among the five elements, the first member of each group being allied to the *yang*,<sup>241</sup> or active principle of the dual philosophy, and the second to the *yin*,<sup>242</sup> or passive principle. For instance, under the element "metal," the stem "weapon"—the "metal" of the warrior—is assigned to the aggressive *yang*, and the stem "cauldron"—the "metal" of the home—to the unassuming *yin*. A similar antonymy prevails among the other stems. The raging "wave" is opposed to the tranquil "stream"; the sturdy "fir" to the slender "bamboo"; the fierce flame of the "torch" to the feeble glimmer of the primitive "lamp"; the rugged "mountain" crag to the orderly cultivated "plain."

There is no particular evidence of a similar division of the American sequence, though it is apparent at a glance



that here are two terms assignable to the aqueous element, "water" (*atl*) and "rain" (*quiahuitl*); three vegetable products, "flower" (*xochitl*), "grass" (*malinalli*), and "cane" (*acatl*.) The "flint" (*tecpatl*) may be considered as a representative of "fire," and the "earthquake" (*ollin*) as a manifestation of earthy influence. *Calli*, "the house," as of mineral origin, may perchance form a second in the "earth" group. There remain "wind" (*ehecatli*) and "death" (*miquiztli*); no great stretch of imagination is required to pair the gentle zephyr and the noxious spirit breath of death in a group ruled by the element "air,"—recognized in Mexican though ignored in Chinese science; and if fancy may be accorded yet further license, and the "cane"—either from its use in blowing fires or as employed in the frictional production of that element—be transferred to the "fire" group, an American series of "elementally-assigned" stems is built up, arranged in groups of twos and exhibiting antonymies roughly corresponding with, and perhaps not more fantastic than those of the Asiatic scheme. "Water," represented by the active "rain," as against the passive "water" in the bowl (cf. the ideogram *atl*). "Wood," the gorgeous "flower" of the cactus offsetting the lowly, every-day "grass." "Earth," in its active manifestation as the all-destroying "earthquake"; passive and immovable, on the other hand, as molded into the vast pile of the *teocalli*, the "house" of the gods. "Air," the fierce all-conquering breath of "death" and the cool refreshing "breeze" of spring. "Fire," the *yang* manifestation produced on the instant from the "flinty rock," and the *yin* avatar of the lordly element brought about by the action of the "cane" spindle of the fire-mill.

Turning now to the animal names, composing the other half of the series, one encounters the "dragon," the "snake," the "rabbit," the "dog," the "monkey," and the "tiger" on both shores of the Pacific. In other words, one

is confronted, as remarked by Gustav Schlegel, by the truly remarkable fact that, of the ten zoological terms of the Mexican cyclical nomenclature, no less than six are absolutely identical with those dedicated to a like purpose in far-away Asia; and since the "eagle" or "vulture" may be accepted without demur as the counterpart of the "cock," the assertion can be made that all but three of the Mexican names find their representatives in the Chinese list. As against the "lizard," "deer," "eagle," and "vulture" of America, there remain the Asiatic "rat," "ox," "horse," "goat" and "pig." The lizard and the deer, creatures very familiar to the Mexicans, may be taken as the analogues of the rat and the ox, (Schlegel equates the lizard with the dragon); the horse and the pig, as animals not indigenous, may reasonably be supposed to have been dropped from the list by those who adapted the series to American use; while the goat, also not a native of Mexico, a rock animal, may have been replaced by the vulture, a bird of the crags.

Such an assignment of the American terms is, no doubt, far-fetched and fantastic,—it is intended as a mere suggestion,—but to those accustomed to follow the history of written characters, of words, of customs, in their migrations from land to land, from people to people, it will rather be matter for surprise that such striking coincidences between the two calendars should be visible on the surface to the casual observer at this day.

In the reassignment of the zoological terms just attempted, the Asiatic and American names were provisionally paired as follows:

(The numerals preceding the names indicate their order in the series.)

## CHINESE.

- |   |       |     |
|---|-------|-----|
| 1 | tzù   | rat |
| 2 | ch'ou | ox  |

## MEXICAN.

- |   |            |        |
|---|------------|--------|
| 4 | cuetzpalin | lizard |
| 7 | mazatl     | deer   |

3	yîn	tiger	14	ocelotl	tiger
4	mào	hare	8	tochtli	hare
5	ch'ên	dragon	1	cipactli	dragon
6	ssú	serpent	5	coatl	serpent
7	wù	horse	—		
8	wéi	goat	15	quauhtli	eagle
9	shen	monkey	11	ozomatli	monkey
10	yù	cock (bird)	16	cozcaquauhtli	vulture (bird)
11	hsü	dog	10	itzquintli	dog
12	hái	boar	—		

Placing now the Maya day-glyphs (corresponding with these Mexican day-names) side by side with the Chinese symbols, thus:

1	tzù			4	kan
2	ch'òu			7	manik
3	yîn			14	ix
4	mào			8	amat
5	ch'ên			1	imix
6	ssú			5	chicchan
7	wù		—	—	
8	wéi			15	men
9	shen			11	chuen
10	yù			16	cib
11	hsü			10	oc
12	hái		—	—	

it will be noticed that:

1. The Maya "grasping hand" sign, *manik*, the seventh American day, finds a place opposite the Chinese "grasping hand," *ch'ou*;

2. Corresponding with the Chinese "tiger" branch, *yin*, comes the Maya *ix*, the fourth day, translated "sorcerer" by Dr. Brinton, a name connected with *balam*, "tiger," and metaphorically "wise man," since the Mayas supposed their sorcerers to possess the power of trans-

forming themselves into tigers. In this connection, the statement found in the Second Appendix to the Chinese Classic of Divination, *Yi-ching*, — that “the great man produces his changes as the tiger does when he changes his spots,” is worthy of notice. Dr. Seler thinks that the Maya glyph *ix* itself “shows the round hairy ear and spotted skin of the jaguar”;

3. The character *mao*, ascribed to the Chinese day of the “hare” is a picture of an “opening door,” typifying the Orient, sunrise, spring. The Maya *katun* corresponding with the Nahuatl day of the rabbit, *tochtli*, is *lamat*, concerning which Dr. Brinton says: “The figures (of *lamat*) bear a close resemblance to some sun signs. . . they seem to show the orb partly below a line, the horizon”;

4. The symbol appertaining to the Chinese “dragon” branch, *ch'en*, is defined by Dr. Wells Williams as “sprouting plants transformed by heaven.” Answering to the Nahuatl “dragon” day, *cipactli*, was the Maya *imix*, which Perez regards as a transposition of *ixim*, “maize.” Again *cipactli* is considered by some writers to denote the “sword-fish” or other marine monster, and Dr. Brinton notes that the head of a fish symbolized the fructifying and motherly waters. One of the Shuo-wen’s explanations of *ch'en*, is, as has been stated above, “pregnant”;

5. The glyph *chicchan*, the fifth day, identified by Seler as a serpent’s skin, corresponds exactly with the Nahuatl *coatl* and Chinese *ssu*, both pictures of snakes;

6. *Chuen*, the eleventh day, the equivalent of the Mexican *ozomatli*, is explained by Brasseur and Seler as “a monkey’s mouth”; it thus corresponds with the Chinese “monkey” branch, *shen*;

7. *Cib*, the symbol of the fourteenth day, is especially noteworthy. It is the correlate of the Nahuatl *cozca-quauhtli*, the “vulture,” and is paralleled by the Chinese day of the “cock,” whose ideogram is *yu*, “a jar of spiritu-

ous liquor," *ci*, "trickling down." The "pottery decoration" (the short parallel strokes several times repeated around the edge of the glyph) certainly indicates the jar or vase." The Tzentel name, *chabin*, for this sixteenth day, has, as some of its meanings, "end," "funereal rites"; with which compare the second form (closing door) of the Chinese character, referred to the death of the day, the setting sun, the fall of the year. The fact that *chab* in the Quiché-Cakchiquel dialects means "arrow" might perhaps suggest a comparison of the Tzentel *chabin* with the tenth Chinese stem *kuei*, said by some to depict an arrow come to rest, and referring, as does the Tzentel word, to the closing scenes of the life-drama.

\* \* \*

Among the Twelve Branches, the ideograms *mao* and *yu*, the opening and closing doors, spring and autumn, east and west, sunrise and sunset, are especially prominent. In the Maya series, three glyphs have been regarded by divers authorities as sun symbols:

Imix	
Lamat	
Akbal	

*Imix* is taken to represent the sun's rays, like the Egyptian hieroglyph *maau*<sup>243</sup> and the Chinese *shih*.<sup>244</sup> *Lamat* is said to picture the disk of the great luminary under some peculiar condition; Dr. Brinton sees in it "the orb partly below a line, the horizon," and from etymological analogies (*lambat*, i. e., "*hundirse in cosa blanda*") explains it as "the sunset." The Chinese pictogram *tan*,<sup>245</sup> presenting the sun and a line, the horizon, has the reverse meaning, "sunrise," and so far as form is concerned may be compared with the Maya *akbal*, said to show the rays

243 

244 

245 

of the sun after sinking below the horizon, and connected with the word *akab*, "night."

Assuming for a moment that the identification of *lamat* as a solar emblem is correct; disposing the series in such a way that this glyph *lamat* comes opposite its Asiatic correlate *yu*, "the closing door"; and ranging the signs which precede and follow the *lamat* glyph in the Maya calendar in the *regular* sequence; thus:

CHINESE.			MAYA.		
1	tzù, child	子	—		
2	ch'òu, hand	手	—		
3	yín, shelter	廄	☉	1	imix, food
4	mào, opening door	𠔁	☽	2	ik, wind
5	ch'ên, containing	𠔁	☽	3	akbal, evening
6	ssú, serpent	巳	☽	4	kan, iguana
7	wù, pestle	杵	☽	5	chicchan, serpent
8	wéi, foliage	𣎵	☽	6	cimi, death
9	shen, stretching	𢇛	☽	7	manik, hand
10	yù, closing door	𠔁	☽	8	<i>lamat</i> , sunset
11	hsū, wounding	𢇛	☽	9	muluc, heap
12	hái, boar	豕	☽	10	oc, dog

We find here that the signs *ik*, *akbal*, *kan*, *chicchan*, *cimi*, *manik*, *lamat*,—"wind," "night sun," "iguana," "serpent," "death," "grasping hand," "sunset," form a series roughly corresponding with the Asiatic sequence of "sunrise," "dragon," "serpent," "pestle," "foliage," "pressing hands," "sunset." If we omit from the Chinese list the "horse" branch, *wu*, whose ideogram is the "pestle,"<sup>246</sup> and which, as before noted, does not seem to have had analogs in the American series, we obtain the following category:

4	sunrise	𠄎		3	sun symbol
5	dragon	辰		4	iguana
6	serpent	巳		5	serpent
7	—	-	-	—	
8	foliage	𣎵		6	death
9	hands	申		7	hand
10	sunset	𠄎		8	sunset

Here the Maya sun symbols are parallel with the Chinese signs of similar import; the iguana and the serpent of the Maya pair off with the dragon and the serpent of the Chinese; and the "hands" of the *shen* branch correspond with the "hand" of the *manik* day. While the symbols of the two series do not form absolutely identical pairs, yet the *relative* positions of the signs in regard to the general series is the *same* in the Chinese list as it was in Mayapan.

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FAIRBANKS, ALASKA.



## THE FRANKLIN SQUARES.

THE following letter with magic squares of  $8 \times 8$  and  $16 \times 16$  are copied from "Letters and papers on Philosophical subjects by Benjamin Franklin, LL. D., F.R.S.," a work which was printed in London, England, in 1769.

FROM BENJAMIN FRANKLIN ESQ. OF PHILADELPHIA,

TO PETER COLLINSON ESQ. AT LONDON.

DEAR SIR:

According to your request I now send you the arithmetical curiosity of which this is the history.

Being one day in the country at the house of our common friend, the late learned Mr. Logan, he showed me a folio French book filled with magic squares, wrote, if I forget not by one Mr. Frenicle, in which he said the author had discovered great ingenuity and dexterity in the management of numbers; and though several other foreigners had distinguished themselves in the same way, he did not recollect that any one Englishman had done anything of the kind remarkable.

I said it was perhaps a mark of the good sense of our mathematicians that they would not spend their time in things that were merely *difficiles nugæ*, incapable of any useful application. He answered that many of the arithmetical or mathematical questions publicly proposed in England were equally trifling and useless. Perhaps the considering and answering such questions, I replied, may not be altogether useless if it produces by practice an habitual readiness and exactness in mathematical disquisitions, which readiness may, on many occasions be of real use. In the same way says he, may the making of these squares be of use. I then confessed to him that in my younger days, having once some leisure

(which I still think I might have employed more usefully) I had amused myself in making these kind of magic squares, and, at length had acquired such a knack at it, that I could fill the cells of any magic square of reasonable size with a series of numbers as fast as I could write them, disposed in such a manner that the sums of every row, horizontal, perpendicular or diagonal, should be equal; but not being satisfied with these, which I looked on as common and easy things, I had imposed on myself more difficult tasks, and succeeded in making other magic squares with a variety of properties, and much more curious. He then showed me several in the same book of an uncommon and more curious kind; but as I thought none of them equal to some I remembered to have made, he desired me to let him see them; and accordingly the next time I visited him, I carried him a square of 8 which I found among my

52	61	4	13	20	29	36	45
14	3	62	51	26	35	30	19
53	60	5	12	21	28	37	44
11	16	50	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	46	25	33	32	17

Fig. 1.

old papers, and which I will now give you with an account of its properties (see Fig. 1). The properties are:

1. That every straight row (horizontal or vertical) of 8 numbers added together, makes 260, and half of each row, half of 260.
2. That the bent row of 8 numbers ascending and descending diagonally, viz., from 16 ascending to 10 and from 23 descending to 17 and every one of its parallel bent rows of 8 numbers make 260, etc., etc. And lastly the four corner numbers with the four middle numbers make 260. So this magical square seems perfect in its kind, but these are not all its properties, there are 5 other curious ones which at some time I will explain to you.

Mr. Logan then showed me an old arithmetical book in quarto,

wrote, I think by one Stifelius, which contained a square of 16 which he said he should imagine to be a work of great labour; but if I forget not, it had only the common properties of making the same sum, viz., 2056 in every row, horizontal, vertical and diagonal.

200	27	252	249	8	25	40	57	72	82	104	121	136	153	168	185
58	39	26	7	250	251	218	189	156	167	154	135	122	103	92	71
198	219	230	251	6	27	33	39	70	91	102	123	134	155	166	187
46	37	26	5	252	229	240	197	138	165	136	133	134	101	92	82
261	216	233	248	9	24	41	56	73	83	105	120	137	152	169	184
55	42	23	16	247	264	215	202	133	170	151	138	119	106	87	74
203	214	265	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	266	213	204	131	172	153	140	117	108	85	76
265	212	257	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	258	241	206	179	174	147	142	115	110	83	78
267	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
43	48	17	16	241	240	269	268	177	176	145	144	113	112	81	80
196	221	225	253	4	29	36	41	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	90	94	67
194	223	226	265	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	266	225	224	193	192	161	160	129	128	97	96	65

Fig. 2.

5	8	9	12
14	15	2	3
11	10	7	6
4	1	16	13

Fig. 3.

Not willing to be outdone by Mr. Stifelius, even in the size of my square, I went home, and made that evening the following magical square of 16 (see Fig. 2) which besides having all the properties of the foregoing square of 8, i. e., it would make 2056 in all the

same rows and diagonals, had this added, that a four-square hole being cut in a piece of paper of such a size as to take in and show through it just 16 of the little squares, when laid on the greater square, the sum of the 16 numbers so appearing through the hole, wherever it was placed on the greater square should likewise make 2056. This I sent to our friend the next morning, who after some days sent it back in a letter with these words:

"I return to thee thy astonishing

"or most stupendous piece

"of the magical square in which"....

—but the compliment is too extravagant and therefore, for his sake, as well as my own I ought not to repeat it. Nor is it necessary,

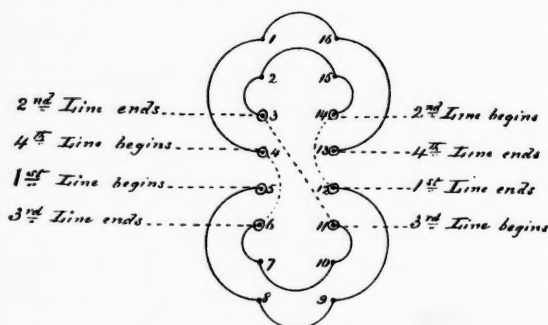


Fig. 4.

for I make no question but you will readily allow the square of 16 to be the most magically magical of any magic square ever made by any magician.

I am etc.

B. F.

It will be seen that the squares shown in Figs. 1 and 2 are not perfect according to the rules of even magic squares previously given, but the interesting feature of their *bent diagonal columns* calls for more than passing notice. In order to facilitate the study of their construction, a  $4 \times 4$  square is given in Fig. 3 which presents similar characteristics.

The dotted lines in this square indicate four bent diag-

onal columns, each of which has a total of 34; three of these columns being intact within the square and one being broken. Four bent diagonal columns may be formed from each of the four sides of the square, but only twelve

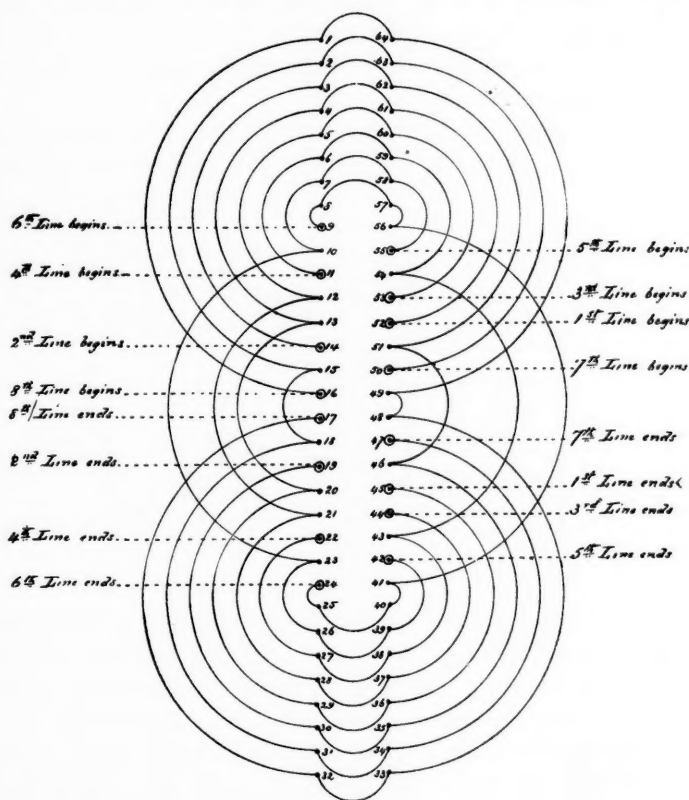


Fig. 5.

of these sixteen columns have the proper totals. Adding to these the eight straight columns, we find that this square contains twenty columns with summations of 34. The  $4 \times 4$  "Jaina" square contains sixteen columns which sum

up to 34 while the ordinary  $4 \times 4$  magic square contains only ten.

The  $8 \times 8$  Franklin square (Fig. 1) contains forty-eight columns which sum up to 260, viz., eight horizontal, eight perpendicular, sixteen bent horizontal diagonals, and sixteen bent perpendicular diagonals, whereas the ordinary  $8 \times 8$  magic square contains only eighteen columns of the same summation.

In addition to the other characteristics mentioned by Franklin in his letter concerning his  $8 \times 8$  magic square

<table> <tr><td>5</td><td>8</td><td>57</td><td>60</td></tr> <tr><td>54</td><td>55</td><td>10</td><td>11</td></tr> <tr><td>43</td><td>42</td><td>23</td><td>22</td></tr> <tr><td>28</td><td>25</td><td>40</td><td>37</td></tr> </table>	5	8	57	60	54	55	10	11	43	42	23	22	28	25	40	37	Section 1. (Top.)	<table> <tr><td>59</td><td>58</td><td>7</td><td>6</td></tr> <tr><td>12</td><td>9</td><td>56</td><td>53</td></tr> <tr><td>21</td><td>24</td><td>41</td><td>44</td></tr> <tr><td>38</td><td>39</td><td>26</td><td>27</td></tr> </table>	59	58	7	6	12	9	56	53	21	24	41	44	38	39	26	27	Section 2.
5	8	57	60																																
54	55	10	11																																
43	42	23	22																																
28	25	40	37																																
59	58	7	6																																
12	9	56	53																																
21	24	41	44																																
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<table> <tr><td>62</td><td>63</td><td>2</td><td>3</td></tr> <tr><td>13</td><td>16</td><td>49</td><td>52</td></tr> <tr><td>20</td><td>17</td><td>48</td><td>45</td></tr> <tr><td>35</td><td>34</td><td>31</td><td>30</td></tr> </table>	62	63	2	3	13	16	49	52	20	17	48	45	35	34	31	30	Section 3.	<table> <tr><td>4</td><td>1</td><td>64</td><td>61</td></tr> <tr><td>51</td><td>50</td><td>15</td><td>14</td></tr> <tr><td>46</td><td>47</td><td>18</td><td>19</td></tr> <tr><td>29</td><td>32</td><td>33</td><td>36</td></tr> </table>	4	1	64	61	51	50	15	14	46	47	18	19	29	32	33	36	Section 4. (Bottom.)
62	63	2	3																																
13	16	49	52																																
20	17	48	45																																
35	34	31	30																																
4	1	64	61																																
51	50	15	14																																
46	47	18	19																																
29	32	33	36																																

Fig. 6.

it may be stated that the sum of the numbers in any  $2 \times 2$  sub-square contained therein is 130, and that the sum of any four numbers that are arranged symmetrically equidistant from the center of the square also equals 130.

In regard to his  $16 \times 16$  square, Franklin states in his letter that the sum of the numbers in any  $4 \times 4$  sub-square contained therein is 2056. The sub-division may indeed be carried still further, for it will be observed that the sum of the numbers in any  $2 \times 2$  sub-square is 514, and there are also other curious features which a little study will disclose.

The Franklin Squares possess a unique and peculiar

symmetry in the arrangement of their numbers which is not clearly observable on their faces, but which is brought out very strikingly in their geometrical diagrams as given

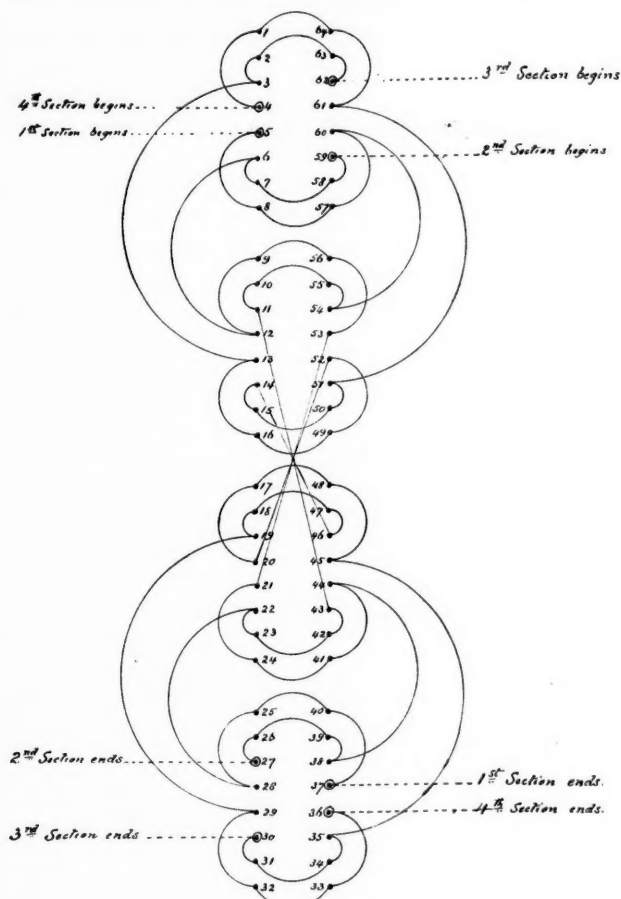


Fig. 7.

in Figs. 4 and 5, which illustrate respectively the diagrams of the  $4 \times 4$  and  $8 \times 8$  squares.



Magic cubes may be readily constructed by expanding these diagrams and writing in the appropriate numbers.

The cube of  $4 \times 4 \times 4$  and its diagram are given as examples in Figs. 6 and 7, and it will be observed that the curious characteristics of the square are carried into the cube.

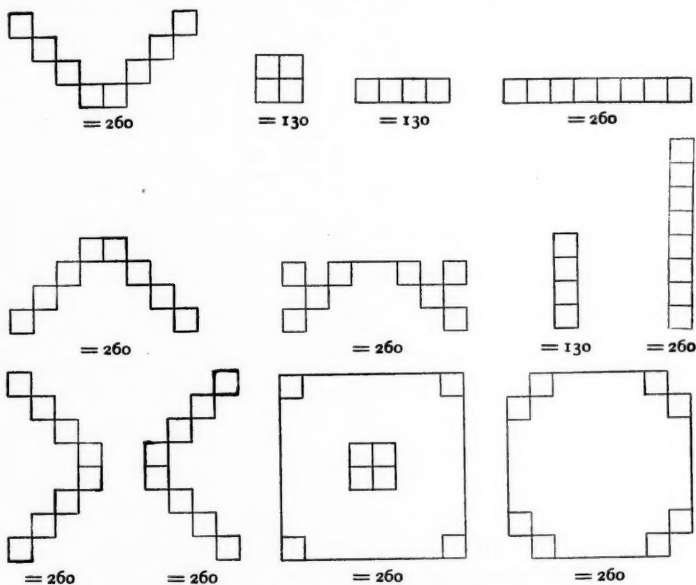
W. S. ANDREWS.

SCHENECTADY, May, 28, 1906.

## CRITICISMS AND DISCUSSIONS.

### THE FRANKLIN SQUARES AND OTHER MATHEMATICAL DIVERSIONS.

The July number of *The Monist* contained an article by C. A. Browne, of New Orleans, on "Magic Squares and Pythagorean



PROPERTIES OF FRANKLIN'S  $8 \times 8$  SQUARE.

Numbers," in which he referred to Benjamin Franklin's interest in the subject. Upon inquiry I found in *The Life and Times of Benjamin Franklin*, by James Parton, (Vol. I, pp. 255-257), a brief

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

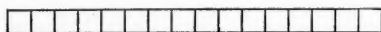
FRANKLIN  $8 \times 8$  SQUARE.

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

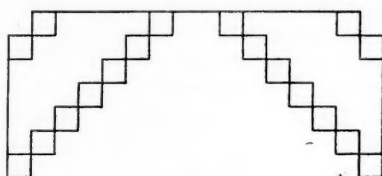
FRANKLIN  $16 \times 16$  SQUARE.



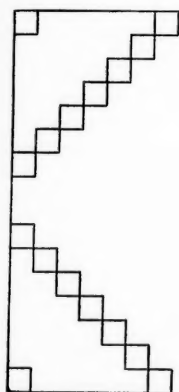
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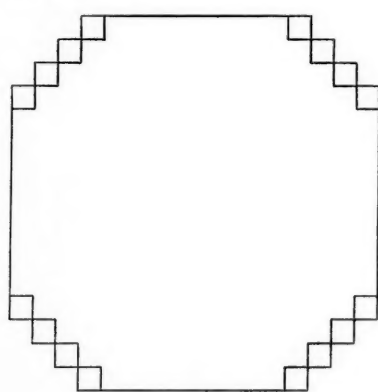
= 2056



= 2056



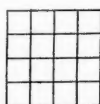
= 2056



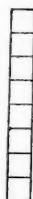
= 2056



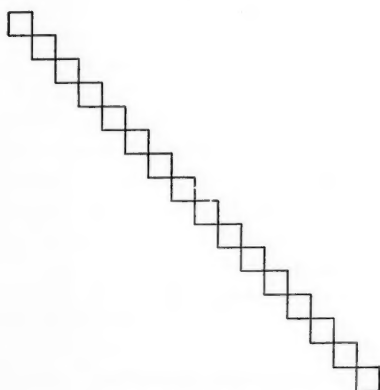
= 2056



= 2056



= 1028



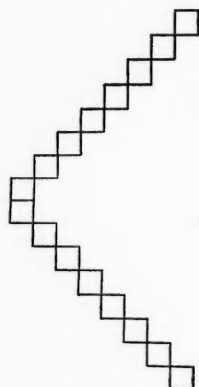
= 2056 - 128

PROPERTIES OF FRANKLIN'S  $16 \times 16$  SQUARE.

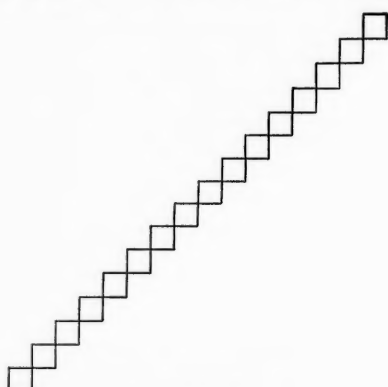
account of two magic squares, one  $8 \times 8$ , the other  $16 \times 16$ , both of which are here reproduced.

Mr. Parton explains the  $8 \times 8$  square as follows:

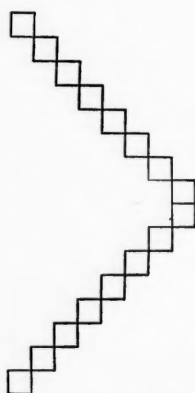
"This square, as explained by its contriver, contains astonishing



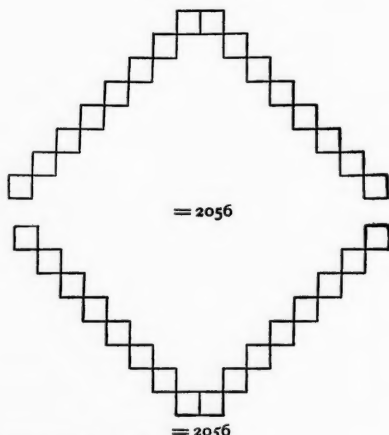
$$= 2056$$



$$= 2056 + 128$$



$$= 2056$$



$$= 2056$$

$$= 2056$$

PROPERTIES OF FRANKLIN'S  $16 \times 16$  SQUARE.

(Continued from preceding page.)

"properties: every straight row (horizontal or vertical) added together makes 260, and each half row half 260. The bent row of eight numbers ascending and descending diagonally, viz., from 16

"ascending to 10, and from 23 descending to 17, and every one of its parallel bent rows of eight numbers, makes 260. Also, the bent row from 52 descending to 54, and from 43 ascending to 45, and every one of its parallel bent rows of eight numbers, makes 260. Also, the bent row from 45 to 43, descending to the left, and from 23 to 17, descending to the right, and every one of its parallel bent rows of eight numbers, makes 260. Also, the bent row from 52 to 54, descending to the right, and from 10 to 16, descending to the left, and every one of its parallel bent rows of eight numbers, makes 260. Also, the parallel bent rows next to the above-mentioned, which are shortened to three numbers ascending and three descending, etc., as from 53 to 4 ascending and from 29 to 44 descending, make, with the two corner numbers, 260. Also, the two numbers, 14, 61, ascending, and 36, 19, descending, with the lower four numbers situated like them, viz., 50, 1, descending, and 32, 47, ascending, makes 260. And, lastly, the four corner numbers, with the four middle numbers, make 260.

"But even these are not all the properties of this marvelous square. Its contriver declared that it has 'five other curious ones,' which he does not explain; but which the ingenious reader may discover if he can."

These remarkable characteristics which Mr. Parton enumerates are illustrated graphically in the accompanying diagrams in which the relative position of the cells containing the numbers which make up the number 260, is indicated by the relation of the small hollow squares.

Franklin's  $16 \times 16$  square is constructed upon the same principle as the smaller, and Mr. Parton continues:

"Nor was this the most wonderful of Franklin's magical squares. He made one of sixteen cells in each row, which besides possessing the properties of the squares given above (the amount, however added, being always 2056), had also this most remarkable peculiarity: a square hole being cut in a piece of paper of such a size as to take in and show through it just sixteen of the little squares, when laid on the greater square, the sum of sixteen numbers, so appearing through the hole, wherever it was placed on the greater square, should likewise make 2056."

The additional peculiarity which Mr. Parton notes of the  $16 \times 16$  square is no more remarkable than the corresponding fact which is true of the smaller square, that the sum of the numbers in any

	1	2	3	4	5	6	7	8	
A	1	2	3	4	5	6	7	8	A
B	9	10	11	12	13	14	15	16	B
C	17	18	19	20	21	22	23	24	C
D	25	26	27	28	29	30	31	32	D
E	33	34	35	36	37	38	39	40	E
F	41	42	43	44	45	46	47	48	F
G	49	50	51	52	53	54	55	56	G
H	57	58	59	60	61	62	63	64	H
	1	2	3	4	5	6	7	8	

THE PLAN OF CONSTRUCTION.

A <sub>1</sub>	B <sub>8</sub>	C	D	E	F	G	H
H <sub>7</sub>	G <sub>2</sub>	F	E	D	C	B	A
3	6						
5	4						
5	4						
3	6						
7	2						
1	8						

First Step.

KEY TO THE SCHEME OF SIMPLE ALTERNATION.

A <sub>1</sub>	B <sub>8</sub>	C <sub>1</sub>	D <sub>8</sub>	E <sub>1</sub>	F <sub>8</sub>	G <sub>1</sub>	H <sub>8</sub>
H <sub>7</sub>	G <sub>2</sub>	F <sub>7</sub>	E <sub>2</sub>	D <sub>7</sub>	C <sub>2</sub>	B <sub>7</sub>	A <sub>2</sub>
A <sub>3</sub>	B <sub>6</sub>	C <sub>3</sub>	D <sub>6</sub>	E <sub>3</sub>	F <sub>6</sub>	G <sub>3</sub>	H <sub>6</sub>
H <sub>5</sub>	G <sub>4</sub>	F <sub>5</sub>	E <sub>4</sub>	D <sub>5</sub>	C <sub>4</sub>	B <sub>5</sub>	A <sub>4</sub>
A <sub>5</sub>	B <sub>4</sub>	C <sub>5</sub>	D <sub>4</sub>	E <sub>5</sub>	F <sub>4</sub>	G <sub>5</sub>	H <sub>4</sub>
H <sub>3</sub>	G <sub>6</sub>	F <sub>3</sub>	E <sub>6</sub>	D <sub>3</sub>	C <sub>6</sub>	B <sub>3</sub>	A <sub>6</sub>
A <sub>7</sub>	B <sub>2</sub>	C <sub>7</sub>	D <sub>2</sub>	E <sub>7</sub>	F <sub>2</sub>	G <sub>7</sub>	H <sub>2</sub>
H <sub>1</sub>	G <sub>8</sub>	F <sub>1</sub>	E <sub>8</sub>	D <sub>1</sub>	C <sub>8</sub>	B <sub>1</sub>	A <sub>8</sub>

Second Step.

COMPLETED SCHEME OF SIMPLE ALTERNATION.

1	16	17	32	33	48	49	64
63	50	47	34	31	18	15	2
3	14	19	30	35	46	51	62
61	52	45	36	29	20	13	4
5	12	21	28	37	44	53	60
59	54	43	38	27	22	11	6
7	10	23	26	39	42	55	58
57	56	41	40	25	24	9	8

Third step.

8×8 MAGIC SQUARE CONSTRUCTED BY SIMPLE ALTERNATION.



$2 \times 2$  combination of its cells yields 130. The properties of the larger square are also graphically represented here.

Franklin's squares were not known to me when I wrote my comments in the January number of *The Monist*; but at the first glance it became obvious that they belong to the same class as the Jaina square quoted by Professor David Eugene Smith (see *Monist* for January, 1906, p. 134), and that the clue to their construction must be sought in the same way.

We write down the numbers in numerical order and call the cells after the precedent of the chess-board, with two sets of symbols, letters and numbers. We call this "the plan of construction."

Before we construct the general scheme of Franklin's square we will build up another magic square, a little less complex in principle, which will be preparatory work for more complicated squares. We will simply intermix the ordinary series of numbers according to a definite rule alternately reversing the letters so that the odd rows are in alphabetical order and the even ones reversed. In order to distribute the numbers in a regular fashion so that no combination of letter and number would occur twice, we start with 1 in the upper left-hand corner and pass consecutively downwards, alternating between the first and second cells in the successive rows, thence ascending by the same method of simple alternation from 1 in the lower left-hand corner. We have now the key to a scheme for the distribution of numbers in an  $8 \times 8$  magic square. It is the first step in the construction of the Franklin  $8 \times 8$  magic square, and we call it "the key to the scheme of simple alternation."

It goes without saying that the effect would be the same if we begin in the same way in the right-hand corners,—only we must beware of a distribution that would occasion repetitions.

To complete the scheme we have to repeat the letters, alternately inverting their order row after row, and the first two given figures must be repeated throughout every row, as they are started. The top and bottom rows will read 1, 8; 1, 8; 1, 8; 1, 8. The second row from the top and also from the bottom will be 7, 2; 7, 2; 7, 2; 7, 2. The third row from the top and bottom will be 3, 6; 3, 6; 3, 6; 3, 6; and the two center rows 5, 4; 5, 4; 5, 4; 5, 4. In every line the sum of two consecutive figures yields 9. This is the second step, yielding the completed scheme of simple alternation.

The square is now produced by substituting for the letter and figure combinations, the corresponding figures according to the consecutive arrangement in the plan of construction.

Trying the results we find that all horizontal rows sum up to 260, while the vertical rows are alternately  $260 - 4$ , and  $260 + 4$ . The diagonal from the upper right to the lower left corner yields a sum of  $260 + 32$ , while the other diagonal from the left upper corner descending to the right lower corner makes  $260 - 32$ . The upper halves of the two diagonals yield 260, and also the sum of the lower halves, and the sum total of both diagonals is accordingly 520 or  $2 \times 260$ . The sum of the two left-hand half diagonals results in  $260 - 16$ , and the sum of the two half diagonals to the right-hand side makes  $260 + 16$ . The sum of the four central cells plus the four extreme corner cells yields also 260.

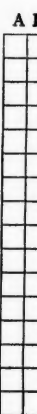
Considering the fact that the figures 1 to 8 of our scheme run up and down in alternate succession, we naturally have an arrangement of figures in which sets of two belong together. This binate peculiarity is evidenced in the result just stated, that the rows yield sums which are the same with an alternate addition and subtraction of an equal amount. So we have a symmetry which is astonishing and might be deemed magical, if it were not a matter of intrinsic necessity.

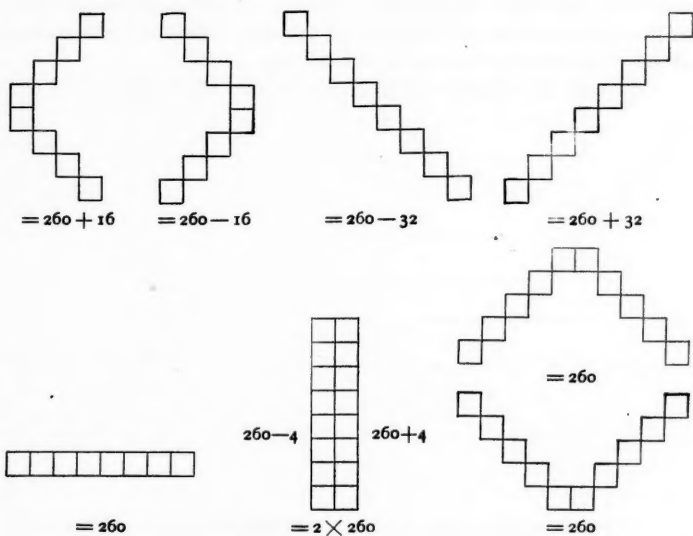
We represent these peculiarities in the adjoined diagrams which, however, by no means exhaust all the possibilities.

We must bear in mind that these magic squares are to be regarded as continuous; that is to say, they are as if their opposite sides in either direction passed over into one another as if they were joined both ways in the shape of a cylinder. In other words when we cross the boundary of the square on the right hand, the first row of cells outside to the right has to be regarded as identical with the first row of cells on the left; and in the same way the uppermost or first horizontal row of cells corresponds to the first row of cells below the bottom row. This remarkable property of the square will bring out some additional peculiarities which mathematicians may easily derive according to general principles; especially what was stated of the sum of the lower and upper half-diagonal of any bent series of cells running staircase fashion either upward or downward to the center, and hence proceeding in the opposite way to the other side.

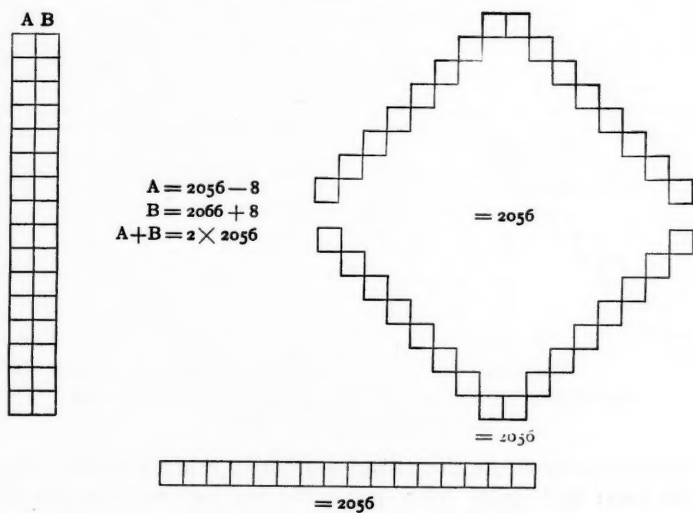
The magic square constructed according to the method of simple alternation of figures is not, however, the square of Benjamin Franklin, but we can easily transform the former into the latter by slight modifications.

We notice that in certain features the sum total of the bent





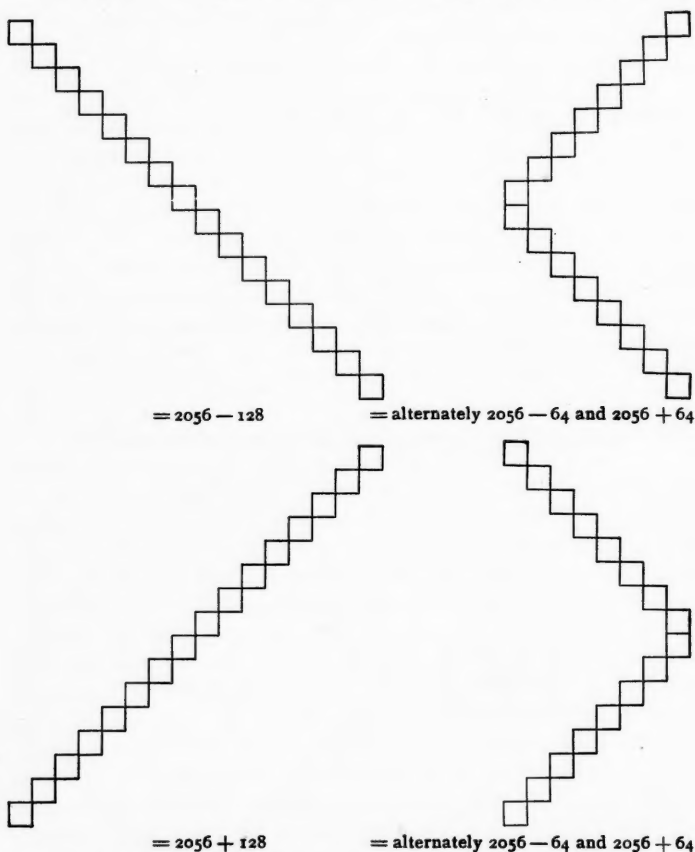
PROPERTIES OF  $8 \times 8$  SQUARE BY SIMPLE ALTERNATION.



PROPERTIES OF  $16 \times 16$  SQUARE BY SIMPLE ALTERNATION.

diagonals represents regularities which counterbalance one another on the right- and the left-hand side. In order to offset these results we have to shift the figures of our scheme.

We take the diagram which forms the key to the scheme of our



PROPERTIES OF  $16 \times 16$  SQUARE BY SIMPLE ALTERNATION.

(Continued from preceding page.)

distribution by simple alternation, and cutting it in the middle, turn the lower half upside down, giving the first two rows as seen in the diagram in which the heavy lines indicate the cutting. Cutting then the upper half in two (i. e., in binate sections), and transposing

1	8
7	2
3	6
5	4
8	1
2	7
6	3
4	5

A <sub>1</sub>	B <sub>8</sub>	C	D	E	F	G	H
H <sub>7</sub>	G <sub>2</sub>	F	E	D	C	B	A
8	1						
2	7						
6	3						
4	5						
3	6						
5	4						

## First Steps.

KEY TO THE SCHEME OF ALTERNATION WITH BINATE TRANSPOSITION.

A <sub>1</sub>	B <sub>8</sub>	C <sub>1</sub>	D <sub>8</sub>	E <sub>1</sub>	F <sub>8</sub>	G <sub>1</sub>	H <sub>8</sub>
H <sub>7</sub>	G <sub>2</sub>	F <sub>7</sub>	E <sub>2</sub>	D <sub>7</sub>	C <sub>2</sub>	B <sub>7</sub>	A <sub>2</sub>
A <sub>8</sub>	B <sub>1</sub>	C <sub>8</sub>	D <sub>1</sub>	E <sub>8</sub>	F <sub>1</sub>	G <sub>8</sub>	H <sub>1</sub>
H <sub>2</sub>	G <sub>7</sub>	F <sub>2</sub>	E <sub>7</sub>	D <sub>2</sub>	C <sub>7</sub>	B <sub>2</sub>	A <sub>7</sub>
A <sub>6</sub>	B <sub>3</sub>	C <sub>6</sub>	D <sub>3</sub>	E <sub>6</sub>	F <sub>3</sub>	G <sub>6</sub>	H <sub>3</sub>
H <sub>4</sub>	G <sub>5</sub>	F <sub>4</sub>	E <sub>5</sub>	D <sub>4</sub>	C <sub>5</sub>	B <sub>4</sub>	A <sub>5</sub>
A <sub>3</sub>	B <sub>6</sub>	C <sub>3</sub>	D <sub>6</sub>	E <sub>3</sub>	F <sub>6</sub>	G <sub>3</sub>	H <sub>6</sub>
H <sub>5</sub>	G <sub>4</sub>	F <sub>5</sub>	E <sub>4</sub>	D <sub>5</sub>	C <sub>4</sub>	B <sub>5</sub>	A <sub>4</sub>

1	16	17	32	33	48	49	64
63	50	47	34	31	18	15	2
8	9	24	25	40	41	56	57
58	55	42	39	26	23	10	7
6	11	22	27	38	43	54	59
60	53	44	37	28	21	12	5
3	14	19	30	35	46	51	62
61	52	45	36	29	20	13	4

## Second Step.

SCHEME OF ALTERNATION WITH  
BINATE TRANSPOSITION.

## Third Step.

SQUARE CONSTRUCTED BY ALTERNA-  
TION WITH BINATE TRANSPOSITION

G <sub>4</sub>	H <sub>5</sub>	A <sub>4</sub>	B <sub>5</sub>	C <sub>4</sub>	D <sub>5</sub>	E <sub>4</sub>	F <sub>5</sub>
B <sub>6</sub>	A <sub>3</sub>	H <sub>6</sub>	G <sub>3</sub>	F <sub>6</sub>	E <sub>3</sub>	D <sub>6</sub>	C <sub>3</sub>
G <sub>5</sub>	H <sub>4</sub>	A <sub>5</sub>	B <sub>4</sub>	C <sub>5</sub>	D <sub>4</sub>	E <sub>5</sub>	F <sub>4</sub>
B <sub>3</sub>	A <sub>6</sub>	H <sub>3</sub>	G <sub>6</sub>	F <sub>3</sub>	E <sub>6</sub>	D <sub>3</sub>	C <sub>6</sub>
G <sub>7</sub>	H <sub>2</sub>	A <sub>7</sub>	B <sub>2</sub>	C <sub>7</sub>	D <sub>2</sub>	E <sub>7</sub>	F <sub>2</sub>
B <sub>1</sub>	A <sub>8</sub>	H <sub>1</sub>	G <sub>8</sub>	F <sub>1</sub>	E <sub>8</sub>	D <sub>1</sub>	C <sub>8</sub>
G <sub>2</sub>	H <sub>7</sub>	A <sub>2</sub>	B <sub>7</sub>	C <sub>2</sub>	D <sub>7</sub>	E <sub>2</sub>	F <sub>7</sub>
B <sub>8</sub>	A <sub>1</sub>	H <sub>8</sub>	G <sub>1</sub>	F <sub>8</sub>	E <sub>1</sub>	D <sub>8</sub>	C <sub>1</sub>

SCHEME OF FRANKLIN'S 8×8 SQUARE.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	A
B	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	B
C	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	C
D	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	D
E	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	E
F	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	F
G	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	G
H	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	H
I	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	I
K	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	K
L	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	L
M	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	M
N	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	N
O	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	O
P	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	P
Q	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	Q
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	

CONSECUTIVE ARRANGEMENT OF NUMBERS IN A 16X16 SQUARE.

A <sub>1</sub>	B <sub>16</sub>	C	D	E	F	G	H	I	K	L	M	N	O	P	Q
Q <sub>15</sub>	P <sub>2</sub>	O	N	M	L	K	I	H	G	F	E	D	C	B	A
3	14														
13	4														
5	12														
11	6														
7	10														
9	8														
9	8														
7	10														
11	6														
5	12														
13	4														
3	14														
15	2														
1	16														

KEY TO THE SCHEME OF SIMPLE ALTERNATION.

A <sub>1</sub>	B <sub>16</sub>	C <sub>1</sub>	D <sub>16</sub>	E <sub>1</sub>	F <sub>16</sub>	G <sub>1</sub>	H <sub>16</sub>	I <sub>1</sub>	K <sub>16</sub>	L <sub>1</sub>	M <sub>16</sub>	N <sub>1</sub>	O <sub>16</sub>	P <sub>1</sub>	Q <sub>16</sub>
Q <sub>15</sub>	P <sub>2</sub>	O <sub>15</sub>	N <sub>2</sub>	M <sub>15</sub>	L <sub>2</sub>	K <sub>15</sub>	I <sub>2</sub>	H <sub>15</sub>	G <sub>2</sub>	F <sub>15</sub>	E <sub>2</sub>	D <sub>15</sub>	C <sub>2</sub>	B <sub>15</sub>	A <sub>2</sub>
A <sub>3</sub>	B <sub>14</sub>	C <sub>3</sub>	D <sub>14</sub>	E <sub>3</sub>	F <sub>14</sub>	G <sub>3</sub>	H <sub>14</sub>	I <sub>3</sub>	K <sub>14</sub>	L <sub>3</sub>	M <sub>14</sub>	N <sub>3</sub>	O <sub>14</sub>	P <sub>3</sub>	Q <sub>14</sub>
Q <sub>13</sub>	P <sub>4</sub>	O <sub>13</sub>	N <sub>4</sub>	M <sub>13</sub>	L <sub>4</sub>	K <sub>13</sub>	I <sub>4</sub>	H <sub>13</sub>	G <sub>4</sub>	F <sub>13</sub>	E <sub>4</sub>	D <sub>13</sub>	C <sub>4</sub>	B <sub>13</sub>	A <sub>4</sub>
A <sub>5</sub>	B <sub>12</sub>	C <sub>5</sub>	D <sub>12</sub>	E <sub>5</sub>	F <sub>12</sub>	G <sub>5</sub>	H <sub>12</sub>	I <sub>5</sub>	K <sub>12</sub>	L <sub>5</sub>	M <sub>12</sub>	N <sub>5</sub>	O <sub>12</sub>	P <sub>5</sub>	Q <sub>12</sub>
Q <sub>11</sub>	P <sub>6</sub>	O <sub>11</sub>	N <sub>6</sub>	M <sub>11</sub>	L <sub>6</sub>	K <sub>11</sub>	I <sub>6</sub>	H <sub>11</sub>	G <sub>6</sub>	F <sub>11</sub>	E <sub>6</sub>	D <sub>11</sub>	C <sub>6</sub>	B <sub>11</sub>	A <sub>6</sub>
A <sub>7</sub>	B <sub>10</sub>	C <sub>7</sub>	D <sub>10</sub>	E <sub>7</sub>	F <sub>10</sub>	G <sub>7</sub>	H <sub>10</sub>	I <sub>7</sub>	K <sub>10</sub>	L <sub>7</sub>	M <sub>10</sub>	N <sub>7</sub>	O <sub>10</sub>	P <sub>7</sub>	Q <sub>10</sub>
Q <sub>9</sub>	P <sub>8</sub>	O <sub>9</sub>	N <sub>8</sub>	M <sub>9</sub>	L <sub>8</sub>	K <sub>9</sub>	I <sub>8</sub>	H <sub>9</sub>	G <sub>8</sub>	F <sub>9</sub>	E <sub>8</sub>	D <sub>9</sub>	C <sub>8</sub>	B <sub>9</sub>	A <sub>8</sub>
A <sub>9</sub>	B <sub>8</sub>	C <sub>9</sub>	D <sub>8</sub>	E <sub>9</sub>	F <sub>8</sub>	G <sub>9</sub>	H <sub>8</sub>	I <sub>9</sub>	K <sub>8</sub>	L <sub>9</sub>	M <sub>8</sub>	N <sub>9</sub>	O <sub>8</sub>	P <sub>9</sub>	Q <sub>8</sub>
Q <sub>7</sub>	P <sub>10</sub>	O <sub>7</sub>	N <sub>10</sub>	M <sub>7</sub>	L <sub>10</sub>	K <sub>7</sub>	I <sub>10</sub>	H <sub>7</sub>	G <sub>10</sub>	F <sub>7</sub>	E <sub>10</sub>	D <sub>7</sub>	C <sub>10</sub>	B <sub>7</sub>	A <sub>10</sub>
A <sub>11</sub>	B <sub>6</sub>	C <sub>11</sub>	D <sub>6</sub>	E <sub>11</sub>	F <sub>6</sub>	G <sub>11</sub>	H <sub>6</sub>	I <sub>11</sub>	K <sub>6</sub>	L <sub>11</sub>	M <sub>6</sub>	N <sub>11</sub>	O <sub>6</sub>	P <sub>11</sub>	Q <sub>6</sub>
Q <sub>5</sub>	P <sub>12</sub>	O <sub>5</sub>	N <sub>12</sub>	M <sub>5</sub>	L <sub>12</sub>	K <sub>5</sub>	I <sub>12</sub>	H <sub>5</sub>	G <sub>12</sub>	F <sub>5</sub>	E <sub>12</sub>	D <sub>5</sub>	C <sub>12</sub>	B <sub>5</sub>	A <sub>12</sub>
A <sub>13</sub>	B <sub>4</sub>	C <sub>13</sub>	D <sub>4</sub>	E <sub>13</sub>	F <sub>4</sub>	G <sub>13</sub>	H <sub>4</sub>	I <sub>13</sub>	K <sub>4</sub>	L <sub>13</sub>	M <sub>4</sub>	N <sub>13</sub>	O <sub>4</sub>	P <sub>13</sub>	Q <sub>4</sub>
Q <sub>3</sub>	P <sub>14</sub>	O <sub>3</sub>	N <sub>14</sub>	M <sub>3</sub>	L <sub>14</sub>	K <sub>3</sub>	I <sub>14</sub>	H <sub>3</sub>	G <sub>14</sub>	F <sub>3</sub>	E <sub>14</sub>	D <sub>3</sub>	C <sub>14</sub>	B <sub>3</sub>	A <sub>14</sub>
A <sub>15</sub>	B <sub>2</sub>	C <sub>15</sub>	D <sub>2</sub>	E <sub>15</sub>	F <sub>2</sub>	G <sub>15</sub>	H <sub>2</sub>	I <sub>15</sub>	K <sub>2</sub>	L <sub>15</sub>	M <sub>2</sub>	N <sub>15</sub>	O <sub>2</sub>	P <sub>15</sub>	Q <sub>2</sub>
Q <sub>1</sub>	P <sub>16</sub>	O <sub>1</sub>	N <sub>16</sub>	M <sub>1</sub>	L <sub>16</sub>	K <sub>1</sub>	I <sub>16</sub>	H <sub>1</sub>	G <sub>16</sub>	F <sub>1</sub>	E <sub>16</sub>	D <sub>1</sub>	C <sub>16</sub>	B <sub>1</sub>	A <sub>16</sub>

SCHEME OF SIMPLE ALTERNATION.

1	32	33	64	65	96	97	128	129	160	161	192	193	224	225	256
255	226	223	194	191	162	159	130	127	98	95	66	63	34	31	2
3	30	35	62	67	94	99	126	131	158	163	190	195	222	227	254
253	228	221	196	189	164	157	132	125	100	93	68	61	36	29	4
5	28	37	60	69	92	101	124	133	156	165	188	197	220	229	252
251	230	219	198	187	166	155	134	123	102	91	70	59	38	27	6
7	26	39	58	71	90	103	122	135	154	167	186	199	218	231	250
249	232	217	200	185	168	153	136	121	104	89	72	57	40	25	8
9	24	41	56	73	88	105	120	137	152	169	184	201	216	233	248
247	234	215	202	183	170	151	138	119	106	87	74	55	42	23	10
11	22	43	54	75	86	107	118	139	150	171	182	203	214	235	246
245	236	213	204	181	172	149	140	117	108	85	76	53	44	21	12
13	20	45	52	77	84	109	116	141	148	173	180	205	212	237	244
243	238	211	206	179	174	147	142	115	110	83	78	51	46	19	14
15	18	47	50	79	82	111	114	143	146	175	178	207	210	239	242
241	240	209	208	177	176	145	144	113	112	81	80	49	48	17	16

16X16 MAGIC SQUARE CONSTRUCTED BY SIMPLE ALTERNATION.



A <sub>1</sub>	B <sub>16</sub>	C	D	E	F	G	H	I	K	L	M	N	O	P	Q
Q <sub>15</sub>	P <sub>2</sub>	O	N	M	L	K	I	H	G	F	E	D	C	B	A
3	14														
13	4														
16	1														
2	15														
14	3														
4	13														
12	5														
6	11														
10	7														
8	9														
5	12														
11	6														
7	10														
9	8														

KEY TO THE SCHEME OF ALTERNATION WITH QUATERNATE TRANSPOSITION.

A SQ

A <sub>1</sub>	B <sub>16</sub>	C <sub>1</sub>	D <sub>16</sub>	E <sub>1</sub>	F <sub>16</sub>	G <sub>1</sub>	H <sub>16</sub>	I <sub>1</sub>	K <sub>16</sub>	L <sub>1</sub>	M <sub>16</sub>	N <sub>1</sub>	O <sub>16</sub>	P <sub>1</sub>	Q <sub>16</sub>
Q <sub>15</sub>	P <sub>2</sub>	Q <sub>15</sub>	N <sub>2</sub>	M <sub>15</sub>	L <sub>2</sub>	K <sub>15</sub>	I <sub>2</sub>	H <sub>15</sub>	O <sub>2</sub>	F <sub>15</sub>	E <sub>2</sub>	D <sub>15</sub>	C <sub>2</sub>	B <sub>15</sub>	A <sub>2</sub>
A <sub>3</sub>	B <sub>14</sub>	C <sub>3</sub>	D <sub>14</sub>	E <sub>3</sub>	F <sub>14</sub>	G <sub>3</sub>	H <sub>14</sub>	I <sub>3</sub>	K <sub>14</sub>	L <sub>3</sub>	M <sub>14</sub>	N <sub>3</sub>	O <sub>14</sub>	P <sub>3</sub>	Q <sub>14</sub>
Q <sub>13</sub>	P <sub>4</sub>	Q <sub>13</sub>	N <sub>4</sub>	M <sub>13</sub>	L <sub>4</sub>	K <sub>13</sub>	I <sub>4</sub>	H <sub>13</sub>	G <sub>4</sub>	F <sub>13</sub>	E <sub>4</sub>	D <sub>13</sub>	C <sub>4</sub>	B <sub>13</sub>	A <sub>4</sub>
A <sub>16</sub>	B <sub>1</sub>	C <sub>16</sub>	D <sub>1</sub>	E <sub>16</sub>	F <sub>1</sub>	G <sub>16</sub>	H <sub>1</sub>	I <sub>16</sub>	K <sub>1</sub>	L <sub>16</sub>	M <sub>1</sub>	N <sub>16</sub>	O <sub>1</sub>	P <sub>16</sub>	Q <sub>1</sub>
Q <sub>2</sub>	P <sub>15</sub>	O <sub>2</sub>	N <sub>15</sub>	M <sub>2</sub>	L <sub>15</sub>	K <sub>2</sub>	I <sub>15</sub>	H <sub>2</sub>	G <sub>15</sub>	F <sub>2</sub>	E <sub>15</sub>	D <sub>2</sub>	C <sub>15</sub>	B <sub>2</sub>	A <sub>15</sub>
A <sub>14</sub>	B <sub>3</sub>	C <sub>14</sub>	D <sub>3</sub>	E <sub>14</sub>	F <sub>3</sub>	G <sub>14</sub>	H <sub>3</sub>	I <sub>14</sub>	K <sub>3</sub>	L <sub>14</sub>	M <sub>3</sub>	N <sub>14</sub>	O <sub>3</sub>	P <sub>14</sub>	Q <sub>3</sub>
Q <sub>4</sub>	P <sub>13</sub>	O <sub>4</sub>	N <sub>13</sub>	M <sub>4</sub>	L <sub>13</sub>	K <sub>4</sub>	I <sub>13</sub>	H <sub>4</sub>	G <sub>13</sub>	F <sub>4</sub>	E <sub>13</sub>	D <sub>4</sub>	C <sub>13</sub>	B <sub>4</sub>	A <sub>13</sub>
A <sub>12</sub>	B <sub>5</sub>	C <sub>12</sub>	D <sub>5</sub>	E <sub>12</sub>	F <sub>5</sub>	G <sub>12</sub>	H <sub>5</sub>	I <sub>12</sub>	K <sub>5</sub>	L <sub>12</sub>	M <sub>5</sub>	N <sub>12</sub>	O <sub>5</sub>	P <sub>12</sub>	Q <sub>5</sub>
Q <sub>6</sub>	P <sub>11</sub>	O <sub>6</sub>	N <sub>11</sub>	M <sub>6</sub>	L <sub>11</sub>	K <sub>6</sub>	I <sub>11</sub>	H <sub>6</sub>	G <sub>11</sub>	F <sub>6</sub>	E <sub>11</sub>	D <sub>6</sub>	C <sub>11</sub>	B <sub>6</sub>	A <sub>11</sub>
A <sub>10</sub>	B <sub>7</sub>	C <sub>10</sub>	D <sub>7</sub>	E <sub>10</sub>	F <sub>7</sub>	G <sub>10</sub>	H <sub>7</sub>	I <sub>10</sub>	K <sub>7</sub>	L <sub>10</sub>	M <sub>7</sub>	N <sub>10</sub>	O <sub>7</sub>	P <sub>10</sub>	Q <sub>7</sub>
Q <sub>8</sub>	P <sub>9</sub>	O <sub>8</sub>	N <sub>9</sub>	M <sub>8</sub>	L <sub>9</sub>	K <sub>8</sub>	I <sub>9</sub>	H <sub>8</sub>	G <sub>9</sub>	F <sub>8</sub>	E <sub>9</sub>	D <sub>8</sub>	C <sub>9</sub>	B <sub>8</sub>	A <sub>9</sub>
A <sub>5</sub>	B <sub>12</sub>	C <sub>5</sub>	D <sub>12</sub>	E <sub>5</sub>	F <sub>12</sub>	G <sub>5</sub>	H <sub>12</sub>	I <sub>5</sub>	K <sub>12</sub>	L <sub>5</sub>	M <sub>12</sub>	N <sub>5</sub>	O <sub>12</sub>	P <sub>5</sub>	Q <sub>12</sub>
Q <sub>11</sub>	P <sub>6</sub>	Q <sub>11</sub>	N <sub>6</sub>	M <sub>11</sub>	L <sub>6</sub>	K <sub>11</sub>	I <sub>6</sub>	H <sub>11</sub>	G <sub>11</sub>	F <sub>11</sub>	E <sub>6</sub>	D <sub>11</sub>	C <sub>6</sub>	B <sub>11</sub>	A <sub>6</sub>
A <sub>7</sub>	B <sub>10</sub>	C <sub>7</sub>	D <sub>10</sub>	E <sub>7</sub>	F <sub>10</sub>	G <sub>7</sub>	H <sub>10</sub>	I <sub>7</sub>	K <sub>10</sub>	L <sub>7</sub>	M <sub>10</sub>	N <sub>7</sub>	O <sub>10</sub>	P <sub>7</sub>	Q <sub>10</sub>
Q <sub>9</sub>	P <sub>8</sub>	O <sub>9</sub>	N <sub>8</sub>	M <sub>9</sub>	L <sub>8</sub>	K <sub>9</sub>	I <sub>8</sub>	H <sub>9</sub>	G <sub>8</sub>	F <sub>9</sub>	E <sub>8</sub>	D <sub>9</sub>	C <sub>8</sub>	B <sub>9</sub>	A <sub>8</sub>

SCHEME OF ALTERNATION WITH QUATERNATE TRANSPOSITION.

1	32	33	64	65	96	97	128	129	160	161	192	193	224	225	256
255	226	223	194	191	162	159	130	127	98	95	66	63	34	31	2
3	30	35	62	67	94	99	126	131	158	163	190	195	222	227	254
253	228	221	196	189	164	157	132	125	100	93	68	61	36	29	4
16	17	48	49	80	81	112	113	144	145	176	177	208	209	240	241
242	239	210	207	178	175	146	143	114	111	82	79	50	47	18	15
14	19	46	51	78	83	110	115	142	147	174	179	206	211	238	243
244	237	212	205	180	173	148	141	116	109	84	77	52	45	20	13
12	21	44	53	76	85	108	117	140	149	172	181	204	213	236	245
246	235	214	203	182	171	150	139	118	107	86	75	54	43	22	11
10	23	42	55	74	87	106	119	138	151	170	183	202	215	234	247
248	233	216	201	184	169	152	137	120	105	88	73	56	41	24	9
5	28	37	60	69	92	101	124	133	156	165	188	197	220	229	252
251	230	219	198	187	166	155	134	123	102	91	70	59	38	27	6
7	26	39	58	71	90	103	122	135	154	167	186	199	218	231	250
249	232	217	200	185	168	153	136	121	104	89	72	57	40	25	8

A SQUARE CONSTRUCTED BY ALTERNATION WITH QUATERNATE TRANSPOSITION.

N <sub>8</sub>	O <sub>9</sub>	P <sub>8</sub>	Q <sub>9</sub>	A <sub>8</sub>	B <sub>9</sub>	C <sub>8</sub>	D <sub>9</sub>	E <sub>8</sub>	F <sub>9</sub>	G <sub>8</sub>	H <sub>9</sub>	I <sub>8</sub>	K <sub>9</sub>	L <sub>8</sub>	M <sub>9</sub>
D <sub>10</sub>	C <sub>7</sub>	B <sub>10</sub>	A <sub>7</sub>	Q <sub>10</sub>	P <sub>7</sub>	O <sub>10</sub>	N <sub>7</sub>	M <sub>10</sub>	L <sub>7</sub>	K <sub>10</sub>	I <sub>7</sub>	H <sub>10</sub>	G <sub>7</sub>	F <sub>10</sub>	E <sub>7</sub>
N <sub>6</sub>	O <sub>11</sub>	P <sub>6</sub>	Q <sub>11</sub>	A <sub>6</sub>	B <sub>11</sub>	C <sub>6</sub>	D <sub>11</sub>	E <sub>6</sub>	F <sub>11</sub>	G <sub>6</sub>	H <sub>11</sub>	I <sub>6</sub>	K <sub>11</sub>	L <sub>6</sub>	M <sub>11</sub>
D <sub>12</sub>	C <sub>5</sub>	B <sub>12</sub>	A <sub>5</sub>	Q <sub>12</sub>	P <sub>5</sub>	O <sub>12</sub>	N <sub>5</sub>	M <sub>12</sub>	L <sub>5</sub>	K <sub>12</sub>	I <sub>5</sub>	H <sub>12</sub>	G <sub>5</sub>	F <sub>12</sub>	E <sub>5</sub>
N <sub>9</sub>	O <sub>8</sub>	P <sub>9</sub>	Q <sub>8</sub>	A <sub>9</sub>	B <sub>8</sub>	C <sub>9</sub>	D <sub>8</sub>	E <sub>9</sub>	F <sub>8</sub>	G <sub>9</sub>	H <sub>8</sub>	I <sub>9</sub>	K <sub>8</sub>	L <sub>9</sub>	M <sub>8</sub>
D <sub>7</sub>	C <sub>10</sub>	B <sub>7</sub>	A <sub>10</sub>	Q <sub>7</sub>	P <sub>10</sub>	O <sub>7</sub>	N <sub>10</sub>	M <sub>7</sub>	L <sub>10</sub>	K <sub>7</sub>	I <sub>10</sub>	H <sub>7</sub>	G <sub>10</sub>	F <sub>7</sub>	E <sub>10</sub>
N <sub>11</sub>	O <sub>6</sub>	P <sub>11</sub>	Q <sub>6</sub>	A <sub>11</sub>	B <sub>6</sub>	C <sub>11</sub>	D <sub>6</sub>	E <sub>11</sub>	F <sub>6</sub>	G <sub>11</sub>	H <sub>6</sub>	I <sub>11</sub>	K <sub>6</sub>	L <sub>11</sub>	M <sub>6</sub>
D <sub>5</sub>	C <sub>12</sub>	B <sub>5</sub>	A <sub>12</sub>	Q <sub>5</sub>	P <sub>12</sub>	O <sub>5</sub>	N <sub>12</sub>	M <sub>5</sub>	L <sub>12</sub>	K <sub>5</sub>	I <sub>12</sub>	H <sub>5</sub>	G <sub>12</sub>	F <sub>5</sub>	E <sub>12</sub>
N <sub>13</sub>	O <sub>4</sub>	P <sub>13</sub>	Q <sub>4</sub>	A <sub>13</sub>	B <sub>4</sub>	C <sub>13</sub>	D <sub>4</sub>	E <sub>13</sub>	F <sub>4</sub>	G <sub>13</sub>	H <sub>4</sub>	I <sub>13</sub>	K <sub>4</sub>	L <sub>13</sub>	M <sub>4</sub>
D <sub>3</sub>	C <sub>14</sub>	B <sub>3</sub>	A <sub>14</sub>	Q <sub>3</sub>	P <sub>14</sub>	O <sub>3</sub>	N <sub>14</sub>	M <sub>3</sub>	L <sub>14</sub>	K <sub>3</sub>	I <sub>14</sub>	H <sub>3</sub>	G <sub>14</sub>	F <sub>3</sub>	E <sub>14</sub>
N <sub>15</sub>	O <sub>2</sub>	P <sub>15</sub>	Q <sub>2</sub>	A <sub>15</sub>	B <sub>2</sub>	C <sub>15</sub>	D <sub>2</sub>	E <sub>15</sub>	F <sub>2</sub>	G <sub>15</sub>	H <sub>2</sub>	I <sub>15</sub>	K <sub>2</sub>	L <sub>15</sub>	M <sub>2</sub>
D <sub>1</sub>	C <sub>16</sub>	B <sub>1</sub>	A <sub>16</sub>	Q <sub>1</sub>	P <sub>16</sub>	O <sub>1</sub>	N <sub>16</sub>	M <sub>1</sub>	L <sub>16</sub>	K <sub>1</sub>	I <sub>16</sub>	H <sub>1</sub>	G <sub>16</sub>	F <sub>1</sub>	E <sub>16</sub>
N <sub>4</sub>	O <sub>13</sub>	P <sub>4</sub>	Q <sub>13</sub>	A <sub>4</sub>	B <sub>13</sub>	C <sub>4</sub>	D <sub>13</sub>	E <sub>4</sub>	F <sub>13</sub>	G <sub>4</sub>	H <sub>13</sub>	I <sub>4</sub>	K <sub>13</sub>	L <sub>4</sub>	M <sub>13</sub>
D <sub>14</sub>	C <sub>3</sub>	B <sub>14</sub>	A <sub>3</sub>	Q <sub>14</sub>	P <sub>3</sub>	O <sub>14</sub>	N <sub>3</sub>	M <sub>14</sub>	L <sub>3</sub>	K <sub>14</sub>	I <sub>3</sub>	H <sub>14</sub>	G <sub>3</sub>	F <sub>14</sub>	E <sub>3</sub>
N <sub>2</sub>	O <sub>15</sub>	P <sub>2</sub>	Q <sub>15</sub>	A <sub>2</sub>	B <sub>15</sub>	C <sub>2</sub>	D <sub>15</sub>	E <sub>2</sub>	F <sub>15</sub>	G <sub>2</sub>	H <sub>15</sub>	I <sub>2</sub>	K <sub>15</sub>	L <sub>2</sub>	M <sub>15</sub>
D <sub>16</sub>	C <sub>1</sub>	B <sub>16</sub>	A <sub>1</sub>	Q <sub>16</sub>	P <sub>1</sub>	O <sub>16</sub>	N <sub>1</sub>	M <sub>16</sub>	L <sub>1</sub>	K <sub>16</sub>	I <sub>1</sub>	H <sub>16</sub>	G <sub>1</sub>	F <sub>16</sub>	E <sub>1</sub>

SCHEME OF FRANKLIN'S 16X16 SQUARE.

the second quarter to the bottom, we have the key to the entire arrangement of figures; in which the alternation starts as in the scheme for simple alternation but skips the four center rows passing from 2 in the second cell of the second row to 3 in the first cell of the seventh, and from 4 in the second cell of the eighth passing to 5 in the first cell, and thence upwards in similar alternation, again passing over the four central rows to the second and ending with 8 in the second cell of the first row. Then the same alternation is produced in the four center rows. It is obvious that this can not start in the first cell as that would duplicate the first row, so we start with 1 in the second cell passing down uninterruptedly to 4 and ascending as before from 5 to 8.

A closer examination will show that the rows are binate, which means in sets of two. The four inner numbers, 3, 4, 5, 6 and the two outer sets of two numbers each, 1, 2 and 7, 8, are brought together thus imparting to the whole square a binate character.

We are now provided with a key to build up a magic square after the pattern of Franklin. We have simply to complete it in the same way as our last square repeating the letters with their order alternately reversed as before, and repeating the figures in each line.

When we insert their figure values we have a square which is not the same as Franklin's, but possesses in principle the same qualities.

To make our  $8 \times 8$  square of binate transposition into the Franklin square we must first take its obverse square; that is to say, we preserve exactly the same order but holding the paper with the figures toward the light we read them off from the obverse side, and then take the mirror picture of the result, holding the mirror on either horizontal side. So far we have still our square with the peculiarities of our scheme, but which lacks one of the incidental characteristics of Franklin's square. We must notice that he makes four cells in both horizontal and vertical directions sum up to 130 which property is necessarily limited only to two sets of four cells in each row. If we write down the sum of  $1+2+3+4+5+6+7+8=2 \times 18$ , we will find that the middle set  $3+4+5+6$  is equal to the rest consisting of the sum of two extremes,  $1+4$ , and  $7+8$ . In this way we cut out in our scheme of symbols, the rows represented by the letters C, D, E, F in either order and accordingly we can shift either of the two first or two last vertical rows to the other side. Franklin did the former, thus beginning

his square with  $G_4$  in the left upper corner. We have indicated this division by heavier lines in both schemes.

The greater square of Franklin, which is  $16 \times 16$ , is made after the same fashion, and the adjoined diagrams will sufficiently explain its construction.

We do not know the method employed by Franklin; we possess only the result, but it is not probable that he derived his square according to the scheme employed here.

Our  $16 \times 16$  square is not exactly the same as the square of Franklin, but it belongs to the same class. Our method gives the key to the construction, and it is understood that the system here represented will allow us to construct many more squares by simply pushing the square beyond its limits into the opposite row which by this move has to be transferred.

There is the same relation between Franklin's  $16 \times 16$  square and our square constructed by alternation with quaternate transposition, that exists between the corresponding  $8 \times 8$  squares.

\* \* \*

Mr. C. A. Browne, Jr. furnishes another interesting square of  $27 \times 27$  representing in addition to its arithmetical qualities commonly possessed by magic squares some ulterior significance of our calendar system referring to the days of the month as well as the days of the year and cycles of years. It is wonderful, and at first sight mystifying, to observe how the course of nature reflects even to intricate details the intrinsic harmony of mathematical relations; and yet when we consider that nature and pure thought are simply the result of conditions first laid down and then consistently carried out in definite functions of a distinct and stable character, we will no longer be puzzled but understand why science is possible, why man's reason contains the clue to many problems of nature and, generally speaking, why reason with all its wealth of *a priori* thoughts can develop at all in a world that at first sight seems to be a mere chaos of particular facts. The purely formal relations of mathematics, materially considered mere nonentities, constitute the bond of union which encompasses the universe, stars as well as motes, the motions of the Milky Way not less than the minute combinations of chemical atoms, and also the construction of pure thought in man's mind.

Mr. Browne's square is of great interest to Greek scholars because it throws light on an obscure passage in Plato's Republic, re-

ferring to a magic square the center of which is 365, the number of days in a year.

The construction of Mr. Browne's square is based upon the simplest square of odd numbers which is  $3 \times 3$ . But it becomes somewhat complicated by being extended to three in the third power which is 27. Odd magic squares are, as we remember, built up by a progression in staircase fashion, but since those numbers

362	381	326	438	468	413	274	303	248	613	642	587	700	729	674	535	564	509	118	147	92	205	234	179	40	69	14
327	353	319	414	440	486	249	275	301	588	614	640	675	701	727	510	536	562	93	119	145	180	206	232	15	41	67
380	325	364	467	412	441	302	247	276	645	586	615	728	673	702	563	508	537	146	91	120	233	178	207	68	13	42
277	306	251	355	384	329	433	462	407	538	567	512	616	645	590	694	723	668	43	72	17	121	50	95	198	228	173
252	278	304	330	356	382	408	434	460	613	539	565	591	617	643	669	695	721	18	44	70	96	122	148	174	200	226
305	250	279	383	328	357	461	405	435	566	511	540	644	589	618	722	667	696	71	16	45	149	94	123	227	172	201
436	465	410	271	300	245	358	387	332	697	726	671	532	561	506	610	648	593	202	231	176	37	66	11	124	153	98
411	437	463	246	272	298	333	359	385	672	698	724	507	533	559	584	620	646	177	203	229	12	38	64	99	125	151
464	409	438	299	244	273	386	331	360	725	670	699	560	505	534	647	592	621	230	175	204	65	10	39	152	97	126
127	156	101	214	243	188	49	78	23	381	390	335	448	477	422	283	312	267	595	624	569	682	711	656	517	546	491
102	128	154	189	215	241	24	50	76	336	362	388	423	449	475	258	284	310	570	596	622	557	683	709	492	518	544
155	100	129	242	187	216	77	22	51	389	334	363	476	421	450	311	256	285	623	568	597	710	655	684	545	490	519
52	81	26	130	159	104	208	237	182	286	315	260	364	393	338	442	471	416	520	549	494	598	627	572	676	705	650
27	53	79	105	131	157	183	209	235	261	287	313	339	365	391	417	443	469	495	521	547	573	599	625	651	677	703
80	25	54	158	103	132	236	181	210	314	259	288	392	337	366	470	415	444	548	493	522	626	571	600	704	649	678
217	240	185	46	75	20	133	162	107	445	474	419	280	309	254	367	396	341	679	708	653	514	543	488	601	630	575
186	212	238	21	47	73	108	134	160	420	446	472	255	281	307	342	368	394	654	680	706	489	515	541	576	602	628
239	184	213	74	19	48	161	106	135	473	418	447	308	253	282	395	340	369	707	652	681	542	487	516	629	574	603
604	633	578	691	720	665	526	555	500	109	138	83	196	225	170	31	60	5	370	396	344	457	486	431	292	321	266
579	605	631	666	692	718	501	527	553	84	110	136	171	197	223	8	32	58	345	371	397	432	458	484	267	293	319
632	577	606	719	664	693	554	489	528	137	82	111	224	169	198	59	4	33	398	343	372	485	430	459	320	285	294
629	558	503	607	636	581	695	714	659	34	63	8	12	141	86	190	219	164	295	324	269	373	402	347	491	480	425
504	530	556	582	608	634	660	686	712	9	35	61	87	113	139	165	191	217	270	296	322	348	374	400	426	452	478
557	502	531	556	580	609	713	658	687	82	7	36	140	85	114	218	163	192	323	268	297	401	346	375	479	424	453
688	717	682	523	552	497	610	638	584	193	222	167	28	57	2	115	144	89	454	483	428	269	318	263	378	405	350
663	689	715	498	524	550	585	611	637	168	194	220	3	29	55	80	116	142	429	455	481	264	290	316	351	377	403
716	661	690	551	496	525	638	583	612	221	166	195	56	1	30	143	88	117	482	427	456	317	282	281	404	349	378

THE MAGIC SQUARE OF MR. C. A. BROWNE.

that fall outside the square have to be transferred to their corresponding places inside, the first and last staircases are changed into the knight's move of the chessboard, and only the middle one retains its original staircase form. We must construct the square so that the central figure, which in a  $3 \times 3$  square is 5, must always fall in the central cell. Accordingly, we must start the square

beginning with figure 1 outside of the square in any middle cell immediately bordering upon it, which gives four starting-points from which we may either proceed from the right or the left, either upwards or downwards which yields eight possibilities of the  $3 \times 3$  square. For the construction of his  $27 \times 27$  square, Mr. Browne might have taken any of these eight possibilities as his pattern. He selected the one starting on the top of the square and moving toward the right, and thus he always follows the peculiar arrangement of this particular square. It is the fourth, in our adjoined diagrams of the  $3 \times 3$  squares. Any one who will take the trouble to trace the regular succession of Mr. Browne's square will find that it is a constant repetition of the knight's move, the staircase move

3			7			1			1		
1	2	7	6	9	1	2	9	4	4	9	2
	9	5	1	9	1	9	5	1	3	5	7
	4	3	8	9	3	7	5	3	7	8	1
	7			3		9			9		
7			3			9			9		
9	8	3	4	1	9	6	7	2	8	1	6
	1	5	9	1	9	1	5	9	3	5	7
	6	7	2	3	7	2	9	4	7	4	9
	3			7		1			1		

THE EIGHT POSSIBLE ARRANGEMENTS OF THE  $3 \times 3$  MAGIC SQUARE.

and again a knight's move on a small scale of  $3 \times 3$  which is repeated on a larger scale  $9 \times 9$ , thus leading to the wonderful regularity which, according to Mr. Browne's interpretation of Plato, astonished the sages of ancient Greece.

Any one who discovers at random some magic square with its immanent harmony of numbers, is naturally impressed by its apparent occult power, and so it happens that they were deemed supernatural and have been called "magic." They seem to be the product of some secret intelligence and to contain a message of ulterior meaning. But if we have the key to their regularity we know that the harmony that pervades them is necessary and intrinsic.

Nor is the regularity limited to magic squares. There are other number combinations which exhibit surprising qualities, and I will here select a few striking cases.

If we write down all the nine figures in ascending and descending order we have a number which is equal to the square of a number consisting of the figure 9 repeated 9 times, divided by the sum of an ascending and descending series of all the figures thus:

$$12345678987654321 = \frac{999999999 \times 999999999}{1+2+3+4+5+6+7+8+9+8+7+6+5+4+3+2+1}.$$

The secret of this mysterious coincidence is that  $11 \times 11 = 121$ ;  $111 \times 111 = 12321$ ;  $1111 \times 1111 = 1234321$ , etc., and a sum of an ascending or a descending series which starts with 1 is always equal to the square of its highest number.  $1+2+1=2 \times 2$ ;  $1+2+3+4+3+2+1=4 \times 4$ , etc., which we will illustrate by one more instance of the same kind, as follows:

$$1234567654321 = \frac{7777777 \times 7777777}{1+2+3+4+5+6+7+6+5+4+3+2+1}.$$

There are more instances of numerical regularities.

All numbers consisting of six equal figures are divisible by 7, and also, as a matter of course, by 3 and 11, as indicated in the following list:

$$\begin{aligned} 111111:7 &= 15873 \\ 222222:7 &= 31746 \\ 333333:7 &= 47619 \\ 444444:7 &= 63492 \\ 555555:7 &= 79365 \\ 666666:7 &= 95238 \\ 777777:7 &= 111111 \\ 888888:7 &= 126984 \\ 999999:7 &= 142857 \end{aligned}$$

Finally we will offer two more strange coincidences of a *lusus numerorum*.

$$\begin{aligned} 0 \times 9 + 1 &= 1 \\ 1 \times 9 + 2 &= 11 \\ 12 \times 9 + 3 &= 111 \\ 123 \times 9 + 4 &= 1111 \\ 1234 \times 9 + 5 &= 11111 \\ 12345 \times 9 + 6 &= 111111 \\ 123456 \times 9 + 7 &= 1111111 \\ 1234567 \times 9 + 8 &= 11111111 \\ 12345678 \times 9 + 9 &= 111111111 \\ 123456789 \times 9 + 10 &= 1111111111. \end{aligned}$$



$$\begin{aligned}
 1 \times 8 + 1 &= 9 \\
 12 \times 8 + 2 &= 98 \\
 123 \times 8 + 3 &= 987 \\
 1234 \times 8 + 4 &= 9876 \\
 12345 \times 8 + 5 &= 98765 \\
 123456 \times 8 + 6 &= 987654 \\
 1234567 \times 8 + 7 &= 9876543 \\
 12345678 \times 8 + 8 &= 98765432 \\
 123456789 \times 8 + 9 &= 987654321.
 \end{aligned}$$

No wonder that such strange regularities impress the human mind. A man who knows only the externality of these results will naturally be inclined toward occultism. The world of numbers as much as the actual universe is full of regularities which can be reduced to definite rules and laws giving us a key that will unlock their mysteries and enable us to predict certain results under definite conditions. Here is the key to the significance of the *a priori*.

Mathematics is a purely mental construction, but its composition is not arbitrary. On the contrary it is tracing the results of our own doings and taking the consequences of the conditions we have created. Though the scope of our imagination with all its possibilities be infinite, the results of our construction are definitely determined as soon as we have laid their foundation, and the actual world is simply one realization of the infinite potentialities of being. Its regularities can be unraveled as surely as the harmonic relations of a magic square.

Facts are just as much determined as our thoughts, and if we can but gain a clue to their formation we can solve the problem of their nature, and are enabled to predict their occurrence and sometimes even to adapt them to our own needs and purposes.

A study of magic squares may have no practical application, but an acquaintance with them will certainly prove useful, if it were merely to gain an insight into the fabric of regularities of any kind.

EDITOR.

## HAECKEL'S THESES. A PROTEST.

Though I feel happy to belong to those of whom Ernst Haeckel says that the clear insight into the evolution of the world not only satisfies the causal feeling of their reason,\* but even their highest emotional cravings, I, on the other hand, quite agree with the great German naturalist that for most educated people of our time monism is destined to be the connecting link between religion and science, and I much appreciate his theses for the organization of monism.

Now I should like to make a few remarks concerning the criticism upon those theses in the January number of *The Monist* (Vol XVI, pp. 120-123), as it has roused my contradiction in several points.

First of all, when looking again through the German text of the theses, I am unable to find that their formulation is "slovenly, and made without due precaution." I should call it clear and uncompromising. And that is what is wanted in our present time, where in most countries a lukewarm tendency is shown in science as well as in politics to reconcile two irreconcilable principles: papism and free thought and teaching.

Further, as regards Kant's doctrine of the *a priori* there is, in my opinion, nothing astonishing that he should reject it, who does not believe in apriorism.

I also can not understand will to be free, when it can not "will as it pleases," and I, therefore, accept determinism in the sense of Haeckel and Paul Rée. Nor can I help, after many years of biological studies, taking it for an established fact that man is nothing else but a social animal most likely descended from one common ancestor with the now living anthropoid apes. When speaking about man, we have not only to think of the highly cultured *homo sapiens* "with all his wealth of moral ideals and scientific attainment," but also of the savage man, who mentally is more closely allied to the ape than, for instance, the critic of Haeckel's theses.

Furthermore, to the best of my knowledge, Ernst Haeckel never disregards in his works the evolutionary phase in religion and science, but he naturally feels no sympathy with the past, where it wilfully hampered the progress of science. The fetishism in Central Africa and in the Celestial Empire did and does, on the

\* Undoubtedly our correspondent here has in mind Schopenhauer's phrase so difficult to translate into English: "Das Causalbedürfniss befriedigen."

other hand, very little, if at all, impede Western civilization, and needed, therefore, not to be mentioned by Professor Haeckel together with the Christian superstitions, nor was there for the same reason any necessity for the courageous philosopher of Jena to denounce the "wild theories" alluded to in the article under review.

PAUL VON RAUTENFELD.

SWATOW, CHINA.

#### EDITORIAL REPLY.

We take pleasure in publishing Herrn von Rautenfeld's protest against our criticism of Professor Haeckel's theses. It proves that our monistic friend of Jena has staunch supporters all over the world, even in Swatow of distant China, but having read Herrn von Rautenfeld's arguments we do not feel compelled to change our views. The Editor of *The Monist* is a personal friend of Professor Haeckel, but the cause of monism would not be well served if for the sake of politeness he would abstain from criticism and avoid controversy.

In former numbers we have discussed our differences with Professor Haeckel, and there is no need of repeating them.\* Be it sufficient here to state that we deem these differences of great importance because their recognition would prevent monism from being narrowed down to a onesided partisan issue, and we feel convinced that Professor Haeckel himself would accept our view if we could have a quiet hour's talk with him.

#### A LETTER FROM PROFESSOR MACH.

The July number of *The Monist* contained an editorial article on "Professor Mach's Philosophy" written in reply to Dr. Hans Kleinpeter's essay "On the Monism of Professor Mach," which appeared in the April number. Dr. Kleinpeter in his enthusiasm for the master whose lead he follows had been carried away in some of his statements to such an extent as to render Professor Mach's

\* "Professor Haeckel's Monism and the Ideas of God and Immortality," *The Open Court*, V, 2957 (Sept. 17, 1891); "Professor Haeckel's Confession of Faith," *The Open Court*, VII, 3529 (Jan. 1893); "Monism not Mechanicism," *The Monist*, II, 438; "Professor Haeckel's Monism," *The Monist*, II, 598; "Theology as a Science," *The Monist*, XIII, 24; "Haeckel's Theses for a Monistic Alliance," *The Monist*, XVI, 120.

views and way of treating problems the final authority in matters philosophical, to the exclusion of all others, criticizing among them also the monism of *The Monist*. The editor not without reluctance undertook to answer Professor Kleinpeter's claims and uphold his own views in face of the new orthodoxy that was put up by this able disciple of one of our most prominent leaders in modern philosophy. We trust that our readers felt that the criticism necessarily implied in our comments should not be taken as any lack of appreciation for a man whose merits in science and philosophy are unquestioned in the world of contemporaneous thought, and we cherish Professor Mach's friendship not only for the sake of his scientific accomplishments, but also for the amiable spirit with which he deals with all questions of a personal nature. Nothing can be more characteristic of the simple modesty of his nature than the letter which he wrote the Editor after the perusal of the two articles. We publish it here in the original German as it would lose many of the fine points in an English translation.

"Hochgeehrter Herr und Freund!

"Die Artikel im *Monist*, April und Juli, habe ich gelesen. Ich bin Ihnen sehr dankbar, dass Sie gedämpft haben. Uebertreibungen können nur unangenehm empfunden werden.

"Ich habe vom Boden meiner Spezialwissenschaft aus einzelne Motive der Methode klar gelegt. Darin bin ich nicht einmal allein, wie ich oft genug hervorgehoben habe. Stallo, Avenarius, Schuppe und wahrscheinlich noch viele andere haben mitgewirkt. Eine abgeschlossene Philosophie habe ich nie gegeben und auch nie beabsichtigt. Darum ist es auch nicht meine Sache, andere Philosophen zu kritisieren. Darum, weil ich einige Sätze vertrete, behaupte ich noch nicht, dass von andern vertretene Thesen wertlos oder unrichtig sind.

"Augenblicklich sehr beschäftigt, sende ich Ihnen eine kleine Ausführung unter Kreuzband, aus welcher Sie sehen, wie wenig ich für mich in Anspruch nehme, und wie sehr ich bemüht bin, den Zusammenhang meiner Gedanken mit älteren aufzuzeigen.

"Einstweilen in ausgezeichnete Hochachtung Ihr ergebenster

"DR. ERNST MACH."

"WIEN, 17 Juli 1906."

There is one point in Professor Mach's letter to which I must take exception, and it is the very thing for which he thanks me so

kindly. Professor Mach is under the impression that I reduce the exaggerated praise of Dr. Kleinpeter, and so he expresses his recognition for the damper I put on it, but this is not so. It was not my intention to detract from Professor Mach's fame and greatness, for I believe that I appreciate his worth as much as does Dr. Kleinpeter; and lest others may have read my article in this sense, I will state here publicly that my admiration for Professor Mach can not be less than that of his most ardent disciple and follower. My intention in writing the article on "Professor Mach's Philosophy" was simply to correct a few errors, or what I deemed to be errors, of Dr. Kleinpeter in his statement of Professor Mach's position, as well as my own relation to it. He did not see that in many points I hold exactly or at least approximately the same views, and it is this kinship between us which has so strongly attracted my attention to Professor Mach's writings. That I do not agree in some other points with Professor Mach is true, but even a statement of that difference will not be regarded as an attempt to lower Professor Mach in the opinion of our readers, or minimize his significance in the scientific world. If Dr. Kleinpeter had made this statement in some other publication, I would not have deemed it necessary to make a reply, but having made it in *The Monist*, it was naturally a puzzle to our readers why I allowed it to go unchallenged, and it appeared like an admission. Moreover, a ventilation of the most important philosophical questions and differences with prominent thinkers can only be beneficial, and we have ventured into the discussion because we were confident that Professor Mach would not misconstrue our answer to Kleinpeter, as implying an attack upon himself.

PAUL CARUS.

### THREE AND A FRACTION.

Prof. Enno Littmann, of Princeton, New Jersey, who has recently been called to the chair of Coptic and Abyssinian at the University of Strassburg, with reference to an editorial article on "The Number  $\pi$  in Christian Prophecy," in the last *Monist*, informs us that there is at present a belief among the Arabs "that the food a man eats stays with him three days and a third."

"A foreigner who comes as a guest to a table (perhaps even a hostile one)," Professor Littmann continues, "is safe for three days and a third after he has eaten from the bread of that tribe. If I am the guest of an Arab tribe and have eaten their food, then

go away, nobody of that tribe may harm me for three and a third days; but after that time even my host may attack and rob me. From this belief also another custom is derived, viz., to let any foreigner stay as a guest for three and a third days.

"When Paul says, 'on the third day,' who knows whether he did not exactly mean Tuesday, for in the Eastern languages of today, Tuesday is called 'Third-day' (as Sunday 'First-day,' Monday 'Second-day,' etc.)? But I do not know just now how old this nomenclature is in the Semite languages. However, 'First-day' for Sunday occurs in the New Testament. By the way the Abyssinians celebrate both Saturday ('small Sabbath') and Sunday ('great Sabbath')."

## BOOK REVIEWS AND NOTES.

THE CODE OF HAMMURABI. By Robert Francis Harper, Ph.D. Chicago: The University Press. Callaghan & Co. London: Luzac & Co., 1904.

All students of Babylonian inscriptions as well as of theology and law will be most grateful to Professor Harper for having given us a book which embodies in it not only a quite careful transliteration and translation of this now most celebrated code of laws, but which gives us also a pretty complete and comprehensive "Index of Subjects," a "List of Proper Names," a "Glossary," and last but not least the "Original" of the code itself. To the "Original" again are added a "List of Signs," a "List of Scribal Errors," a "List of Erasures," and a map of Hither Asia. The student thus finds in this volume all that is necessary for a deeper study and appreciation of this venerable document.

We must not, however, expect the book before us to be a *final* edition of either the original text itself or of its transliteration or translation. Says Professor Harper himself: "The text as represented in Plates I—LXXXII has been reconstructed and edited from the photographs published by Scheil in Tome IV, *Textes Elamites-Sémitiques* of the *Mémoires de la Délégation en Perse*, Paris: Leroux, 1902;" and again: "I am of opinion that an edition of an Assyrian or Babylonian text which is to be final must go back to the originals." (Introduction, pp. xii, xiii.) All that this edition is intended for is to put into the hands of students the laws of Hammurabi in a convenient and handy form. That Professor Harper has succeeded in his task most admirably nobody can or will deny; and I am certain all students will be grateful to him for such a convenient edition.

It is not my intention to go into a detailed criticism of this useful book—that would be out of place and would lead too far. A few remarks, however, may be permitted here:

The transcription is not carried through according to uniform rules; thus e. g. there is no distinction made between the different kinds of *u* or *shu* or *gi*, etc., etc., nor is the same word uniformly transcribed, as e. g. *ilu* EN-LIL I, 3. 46. 53 etc., but *ilu Bêl* in XL, 12. 64; XLII, 53. 89. 95; XLIV, 86 and *ilu bêlu-ut* for *ilu* EN-LIL-ut I, ii. Sometimes we have two different kinds of transcription for the same word, thus *Dur-ilu-KI* in I, 59 is given as *Dûr-ilu* in the List of Proper Names, where it is said to be a city! This, no doubt, must be a printer's error, for surely Professor Harper would know that *Dûr-ilu* the city where god Kadi was supreme (*Early Babyl. Hist.*, p. 255) is written quite differently, viz. BAD=*dûru* wall, while we have here the

sign DUR=*riksu*=band. This name must be transcribed not *Dur-ilu-KI* but *Dur-an-ki*; nor is it a city, but one of the names of the zigurrat of *E-kur*, the temple of Enlil of Nippur. (See my "Bel, the Christ of Ancient Times," in *The Monist*, Vol. XIV, No. 1, p. 87.) Hence, also, we cannot translate col. I, 59 by "for Nippur and Durilu" but by "for the Nippurian Dur-an-ki." For this translation compare the "Nippurian Bêl" in *Early Babylonian History*, p. 271, 5 and p. 272, 6. The occurrence of the "Nippurian Dur-an-ki" in this code proves that at the time of Hammurabi there already existed another Dur-an-ki, viz., that at Sippar (cf. the king of Sippar: *Enme-Duranki*, Zimmern, KAT<sup>3</sup>, p. 532) as well as that at Larsa (IV R 24, 50a; II R 58 No. 3, 25).

Enlil as "god of Dur-an-ki" is "god of the band of heaven and earth" (II R 54, 4a) and as such the "father of the gods" (K. B. VI<sup>1</sup> 46, 8), Shamash as the *En-me-* (i. e., *Hauptbefehlsvollstrecker* or "lord of commands") of *-dur-an-ki* is his "chief messenger," as such having the rôle of Nin-Girsu, the מלאך יהוה. Therefore is Shamash called the *ilu Ud-gishgal-lu* (IV R 19, 47, 48) i. e., the *Wolkensturm, finsterer Sturm* (Del. Hwb. p. 136 sub *irpû*)—a name attributed not only to Nergal (Shalmanassar II, Bronze Gates from Balawat, col. V, 4, 5), but also to NIN.IB=Nin-Girsu (II R 57, 51c).

In the "List of Proper Names" we miss badly the determinative "*alu*" before certain names as e. g. *alu NUN-KI* I, 64; *alu Shesh-ab-ki* II, 17, but *Ud-kib-nun-ki* without *alu* II, 25!

Likewise the determinative *ilu* ought to have been given in every case, and where it has been omitted (e. g. U.DAR: III, 54; IV, 47; I-gi-gi I, 14) this ought to have been indicated.

Also in the Glossary the transcription does not harmonize with that given in the text, as. e. g., *shu-bi-i-im* occurs correctly in the text I, 34, but in the Glossary it is given as *shu-be-i-im* etc., etc.

In the Introduction, p. xiv, we are told "the sign under No. 121 (sic! read 125), which has not been explained hitherto, has been made a *gunu* of No. 148" (i. e., UD). The sign in question is by no means new. Thureau-Daugin already in his *Récherches sur l'origine de l'écriture cunéiforme* sub No. 92 said of that sign "le signe paraît être le *gunû* de UD." An older form of this very sign occurs also in my *Early Babylonian History*, p. 428, seal I. 3: *Ur-ud-azag-ga!*

How one may be of quite a different opinion as to the meaning, understanding, transcription and translation of certain portions of this law may be illustrated by the first column of the Epilogue. I select just this portion because it is most important not only for forming a correct opinion of the religious teaching at the time of Hammurabi, but also because it throws a new and welcome light on the much disputed and vexed question as to whether there was in the Babylonian pantheon a highest god called *ilu* "god."

For the sake of convenience I shall put Professor Harper's and our translation of col. I side by side.

Professor Harper translates:

1. When the lofty Anu,
2. king of the Anunnaki,
3. and Bel,

1. When the exalted GOD,
2. the king of the (ilu) A-nun-na-ki,
3. (ilu) Enlil,



4. lord of heaven
5. and earth,
6. he who determines
7. the destiny of the land,
13. committed
11. the rule
12. of all mankind
8. to Marduk,
9. the chief son
10. of Ea;
15. when they made him great
14. among the Igigi;
17. when they pronounced the lofty name
16. of Babylon;
19. when they made it famous
18. among the quarters of the world
20. and in its midst
26. established
21. an everlasting kingdom
24. whose foundations
25. were firm
22. as heaven
23. and earth
27. —at that time,
45. Anu
46. and Bel
49. called me,
28. Hammurabi,
30. the exalted
29. prince,
31. the worshiper of the gods,
32. to cause justice
34. to prevail
33. in the land,
36. to destroy
35. the wicked and the evil,
39. to prevent
37. the strong 39 from oppressing
38. the weak,
42. to go forth
40. like the sun
41. over the Black Head Race,
44. to enlighten
43. the land
48. and to further
47. the welfare of the people.
50. Hammurabi,
4. the lord of heaven
5. and earth,
6. who determines
7. the destiny of mankind,
8. to (ilu) Amar-ud (Marduk),
9. the principal son
10. of (ilu) En-ki (Ea),
11. the (ilu) Enlil-ship (lordship)
12. over the totality of men
13. had committed to him;
14. had among the I-gi-gi
15. made him great;
16. had of Babylon
17. its lofty name pronounced;
18. had in the world-regions
19. made it (=Babylon) famous;
20. had in its (=Babylon's) midst
21. an everlasting kingdom
22. whose
24. foundations 22 are as heaven
23. and earth
25. firmly founded,
26. for him (=Marduk) established
27. —then at that time:
28. Hammurabi,
29. the prince,
30. the exalted
31. who reverences the gods: me,
32. for the purpose of making justice
33. in the land
34. to shine forth, and
36. of destroying
35. the wicked and the evil, and
37. of preventing the strong
39. from oppressing
38. the weak,
42. to go forth
40. like (ilu) Ud (Shamash)
41. over the Black-Heads( =man-kind),
44. to enlighten
43. the land,
45. GOD,
46. viz., (ilu) Enlil,
48. to further
47. the bodily welfare of the people,
49. called by name.
50. Hammurabi,

- |                                             |                                          |
|---------------------------------------------|------------------------------------------|
| 51. the governor                            | 51. the shepherd,                        |
| 52. named                                   | 52. the called one                       |
| 53. by Bel, am I,                           | 53. by (ilu) Enlil, am I,                |
| 54. who brought about                       | 54. who gave in its completeness         |
| 55. plenty                                  | 55. plenty                               |
| 56. and abundance;                          | 56. and abundance;                       |
| 57. who made                                | 57. who made                             |
| 58. everything                              | 58. everything                           |
| 59. for Nippur and Durilu 57 com-<br>plete; | 59. for Nippur-Duranki 57 com-<br>plete; |
| 61. the exalted                             | 61. the exalted                          |
| 60. supporter                               | 60. beautifier                           |
| 62. of E-kur;                               | 62. of E-kur,                            |
| 63. the wise king,                          | 63. the strong king,                     |
| 64. who restored Eridu                      | 64. 65. who rebuilt Eridu; etc.          |
| 65. to its place; etc.                      |                                          |

The sense of this column is the following:

When GOD (1) Enlil (3) had committed (13) to Marduk (8) the Enlilship over the totality of men (11-12) and had done all that follows in ll. 14-26 then at that time (27) GOD (45) i. e. Enlil (46) in order to make the people fare well (47-48) called by name (49) Hammurabi (28): ME (31) to go forth like Shamash (40-44) with the avowed purpose of doing what follows in lines 32-39. Hammurabi, who wrote or had this code written, speaks of himself in lines 1-30 in the third person and from line 31ff in the first—therefore “me.”

In justification of my translation and in order to show that there was at the time of Hammurabi a *highest god* called AN=*ilu*, with whom Dagan, Enlil, Sin, IB etc. had been or were in course of time identified, I may be permitted to give the following note here.

The god Anu, whom Harper finds in line 1, is not mentioned by one syllable in this paragraph—hence the correct transcription (n)*i-nu ilum si-ru-um* is wrongly translated by “when the lofty Anu.” In line 3 there is in the original no “and” before *ilu En-lil*—and if not, then lines 1, 2 can only be an *emphatic* apposition (as such standing at the beginning, *before* its noun!) to *ilu En-lil*, and have to be translated by “when the lofty (or exalted) GOD, the king of the Anunnaki: Enlil, lord of heaven and earth etc.” That this is the only correct translation follows from the following considerations:

1. Marduk receives in line 1 from AN=*ilum* and in line 3 from (*ilu*) *En-lil* the (*ilu*) *En-lil-ut* (line 11), i. e., the Enlilship. But the Enlilship can be given away by Enlil *only*, and not by Anu and Enlil. Anu can dispose of the (*ilu*) *A-nu-ut*, i. e., the Anuship, but never of the Enlilship.
2. Such *uncontracted* forms as *u-shar-be-u-shu* (line 15); *ib-bi-u* (line 17) show us that they are *singulars* (therefore *uncontracted*; if they were plurals they would be contracted!) Against this is not to be brought in the form *ib-bu-u* (line 49), which is a *pausal* (therefore *contracted*) form in the *singular*, and not a plural.
3. Hammurabi tells us in lines 52, 53 that he is the *ni-bi-it ilu En-lil*,

i. e., the called (one) by Enlil. Notice, he does not say here that he is called by Anu and Enlil! Hence Enlil, and Enlil only, called him. But if Enlil and Enlil only called him, then the *ilum* (AN) *u* (*ilu*) *En-lil shu-mi ib-bu-u* in lines 45ff. can only be translated by: "(at that time) GOD, i. e., Enlil called (me) by name."

The translation of this passage, then, hinges on the right understanding of the *u*="i. e." Are there other passages in the Cuneiform literature which likewise establish for this *u* (generally written SHI + LU) the meaning "i. e." or "viz."? Quite a number of passages might be brought in here. For our purpose, however, the following will suffice: In Tiglath-Pileser I, Prisma, col. V, 29ff. we read:

*mâtâte* (kur-kur-mesh) *Na-i-ri* 30 *rapshâte* (mesh-te) *a-na pat gim-ri-shi-na a-bil* 31 *u* (=SHI + LU) *nap-char sharrâni* (mesh-ni)-*shu-nu* 32 *a-na shepê* (mesh)-*ia u-kin-ni-ish*, i. e., the extended Nairi-lands in their totality I subdued, i. e., the totality of their kings I put under my feet.

Among Harper's own publications, *Letters of the K. Collection*, Vol. III, p. 276, No. 274=K. 81, Obv. ll. 8 ff. we read:

*AN* (mesh) *GAL* (mesh) *sha AN-e ù KI-tim a-na LUGAL EN-ia lik-tar-ra-bu ù* (=SHI+LU) (*gish*) *GU.ZA LUGAL EN-ia libbu (bu)-ù AN-e a-na da-resh lu-kin-nu*, i. e., "may the great gods of heaven and earth bless the king, my lord, i. e., may they establish the throne of the king, my lord, in the midst of the world (sic!) for all eternity!" The blessing invoked upon the king consists in the establishment of an eternal reign—the latter sentence, then, explains the former!

The *u* in both cases cannot be translated by "and," for if "and" were intended then the Assyrian had to be according to Grammar ="-ma."

This *u explicativum*, as we may call it, in the sense of "i. e.," "namely," "viz.," is especially frequent in such connections as:

*AN u(ilu) Dagan*, and *AN u(ilu) Enlil*, etc., etc.

On the basis of this explanation two passages in our code receive a new light:

1. We know that Babylon was the city of Marduk; we also know that *AN si-ru-um* (*ilu*) *En-lil* (I, 1-3) committed to Marduk the Enlilship over the totality of men (I, 11, 12), i. e., Enlil transfers his rôle to Marduk (conf. XL, 9 ff.: "The black-head people, whose rule Enlil presented and Marduk delivered to me," i. e., from Enlil through Marduk to Hammurabi!) who now becomes the *An* (*ilu*) *En-lil* or *AN u(ilu) En-lil*. It is, therefore, possible that Babylon might be called also "the city of AN *u(ilu) En-lil*." Indeed so it is actually called in our code, col. XL, 63, 64: *i-na KA-DINGIR-RA-KI alim sha AN u* (*ilu*) *En-lil*, which, however, cannot be translated by "in Babylon, the city whose turrets Anu and Bel raised," but which must be rendered by "in Babylon, the city whose turrets GOD, i. e., the Lord (Marduk) raised." (Harper completely mistranslates this passage.)

2. In our code, col. II, 47, we read:

*a-na AN+NIM u* (=SHI+LU) *ilu Nanâ*.

Harper transcribes here: *a-na ilu Anu u ilu Nanâ*—but *AN+NIM* can never be read *ilu Anu*, nor *Anim* (Winckler), which latter ought to be at least *A-nim* and not *AN-nim*. The sign NIM has also the pronunciation "tum" and *AN+NIM* can be read only either *An-tum* or *il-tum*. *An-tum*

u *ilu Nanā* is "the goddess, viz., Nanā." That the goddess Nanā or Ishtar was indeed identified with An-tum, the goddess *καρ' έλοχην*, is evident from II R 54, n. 3, 19. The same is true of XLIV, 15 where Ninkarrak is called the daughter of AN+NIM=An-tum.

These observations are, as was indicated above, of the highest importance for a right understanding of the much disputed question as to whether the Babylonians had a highest god called "*ilu*." Says Zimmern, KAT<sup>1</sup>, p. 354: "Einen babylonischen Obergott *Ilu*, der in früheren Darstellungen der babylonischen Mythologie eine ziemliche Rolle spielte, giebt es nicht; vielmehr handelt es sich in den Fällen, wo *ilu* in den babylonischen Götterlisten aufgeführt wird, einfach um die appellativische Bezeichnung *ilu* 'Gott.'"

This cannot be maintained any more now. On the contrary, we have to say:

The highest honor a god or goddess could receive was to be identified with AN = *ilu* or *iltum*, which AN in such cases is not a mere appellative nor "determinative" but which shows us that the god who bears this title was now considered to be equal to that old god AN the "totality of heaven and earth," like unto whom there was no other god—who, therefore, stands in all the "lists of gods" at the very top. AN, although originally the god (*ilu*) was yet later on displaced by En-lil, the "lord of the wind." By thus displacing AN, Enlil acquired in addition to his old attributes (lord of the wind) also those of AN, i. e., he became the "god of heaven and earth," the god *καρ' έλοχην*. At this time (i. e., when Enlil had become the god of heaven and earth or what is the same, the god of the whole of Babylonia) it seems that the oldest Sumerian cosmology and theology was developed: Enlil being now = "heaven and earth," AN (together with his brother KI) were made to be the "upper or heavenly" and the "lower or terrestrial" ocean. The reminiscence, however, continued to live on that AN was the *first* and *oldest* god of "heaven and earth"—hence the desire of the Babylonian theologians to identify their most favored god who happened to be, at one time or another, the most influential god of Babylonia, or to express it somewhat differently, to identify that god, whose city happened to be the capital of Babylonia, with the first and oldest god of the Babylonian "high and low-land": with AN the *ilu καρ' έλοχην*, thus claiming for their god the necessary authority to exercise his power.

Again when Enlil was displaced by Marduk it is natural that he likewise should be called the AN u (*ilu*) Enlil, "the GOD, the LORD," thus having received his authority from AN through Enlil.

In this wise not only Enlil, Marduk, Sin, IB etc. were identified with AN, but even their respective wives had to become likewise the An-tum or il-tum *καρ' έλοχην*, thus Ninlil, Sarpanitum (=Ishtar), Ningal, NIN-IB, Ishtar—all were=Antum or il-tum the goddess at one time or another.

That *ilu* was indeed the name of the highest god, i. e., of the god who happened to be the god of the capital of Babylonia is evident also from the following:

1. The celebrated trilingual list of gods, II R 59, gives as the first and foremost god, who stands at the head and who precedes even Enlil:

*dim-me-ir* | AN | *i-[lum]*.

This very same AN is, in other lists of gods, said to be pronounced

A-nu-um (II R 54, No. 3; III R 69 No. 1; II R 54 No. 4)—hence the Sumerian AN had as “god” the pronunciation A-nu-um or i-lum. From this it follows that the god “ilum” is neither the Canaanitish El (as Winckler wants), nor is the word A-nu-um a pure Babylonian word. On the contrary, we will have to admit that the Semitic-Babylonian name for the Sumerian god AN was “ilu” and that the word A-nu-um is the Semiticized Sumerian pronunciation of AN, i. e., Anum is the Sumerian AN + Semitic nominative ending “u” + mimmatum “m.”

2. The name of “Babylon” is generally written

*Bâb-ili-ki* or

*Bâb-DINGIR-RA-ki* or

*Bâb-ili(mesh)-ki*

*Bâb-ilu-ilu-ki* (Nebk. E. I. H., IV, 32).

*Ba-ab-NI-NI*. (K. 4309, 22 = Str. List 5997).

and is most commonly translated by “gate of god;” a translation which may mean either (a) *gate*, which god is (cf. Christ’s “I am the door”) genetivus subjectivus; (b) *gate*, which leads to god (cf. Christ’s “by me if any one enters in he shall be saved”) genetivus objectivus. And as Marduk was the god of Babylon it is natural to suppose that the “god” referred to in “gate of god” was Marduk himself. Babylon, however, was in existence before Marduk appeared in Babylonia. We may, therefore, grant the possibility that by this “god” in “gate of god” was meant originally another “god”—the god namely who was at one time or another (before Marduk appeared) the god of Babylonia as a whole. Furthermore, as Hammurabi is very anxious to identify Marduk with AN *u(ilu)Enlil*; he calls Babylon “the city whose turrets AN *u(ilu)Enlil* raised”—gods of whom we know that they actually ruled over the whole of Babylonia—we may conclude that the *ilu* in *bâb-ilu* signifies always the *highest* god: may the highest god have been AN, i. e., *ilu*, or Marduk or Enlil, therefore also the pluralis majesticus *Bâb-ili(mesh)-ki* or *Bâb-ilu-ilu-ki* or *Ba-ab-NI-NI*! For *NI-NI=i-li*=plural (of majesty) see XLII, 45: AN *ra-bu-um a-bu NI-NI*, i. e., “the great *ilu*, the father of the gods.”

This is important for understanding the “proper names” composed with *ili* (mesh), *ilu-ilu*, or *NI-NI*. In such names the *ili* (mesh) etc. is simply the *highest god*—whether he was considered to be Marduk or Enlil or Sin etc. does not matter. Now we also understand the writing *Bâb-(ilu)KUR-GAL-ki*=Babylon (II R 57 Rev. line 70). *(ilu)KUR-GAL* is according to V R 44 III, 41 =Enlil resp. Bêl. *Bâb-(ilu)KUR-GAL-ki*, then, may be read *Bâb-Bêl*: the gate of Bêl—a reading which would show us that the “*ilu*” at one time or another was the old Enlil—hence Babylon “the city of GOD, i. e., Enlil”! But the name also demonstrates beyond the shadow of a doubt that Enlil was *ilu*, the god *kar’ êtoχn* and that *Bâb-(ilu)KUR-GAL-ki* may therefore be read also *Bâb-ili-ki*. *(ilu)KUR-GAL*, because = *ilu* Enlil, is also = *ilu E-kur*. But *ilu E-kur* is = A-nu-um resp. *ilum*. (See my “Bel, the Christ of Ancient Times,” *The Monist*, Vol. XIV, No. 1, p. 83, Note 6)—hence *ilu KUR-GAL* = *ilu(m)*, and *Bâb-(ilu)KUR-GAL-ki* = *Bâb-ili-ki*.

In conclusion I should like to rectify two minor mistakes of Professor Harper:

The word *i-nu-mi-shu* in I, 27 is registered in the Glossary, p. 153, as a word by itself—as if there existed in Babylonian an *inumi*! This is wrong! *i-nu-mi-shu* is contracted out of *ina ûmi-shu*. For such contractions with *ina* cf. among other places also K8531, Rev. 4: *ina nu-ush-shu* where the duplicate Rm. 126, Rev. 4, has *i-nu-ush-shu*. Hrozný's Grammar, *Mythen von dem Gotte Ninrag*, p. 48, is likewise not what it ought to be, for he takes *inushshu* to be a Present tense!

The expression "and to further the welfare of the people" (I, 47, 48) does not belong to Hammurabi, but to AN u (ilu) Enlil, therefore the "and," which is not found in the original is also not necessary here.

HUGO RADAU.

CHICAGO, NOV., 1904.

P. S. Since the above was written there appeared in the A. J. S. L. L., Vol. XXII (October, 1905) Professor Harper's Commentary to the Code of Hammurabi,—which, however, contains no corrections, and Harper tells us, for instance, (p. 145) that Ninazu is a goddess. Apparently he has not seen such passages as II R 18, 40 = A. S. K. T., No. 11, col. iv, 40.41:

"(*dingir*) *Nin-ki-gal dam (dingir) Nin-a-zu-ge | (ilu) ditto al-ti (ilu) ditto*," i. e., "Ninkigal the wife of Ninazu"—hence the latter must be the husband, i. e., he must be a male god! Cf. also II R 59 Rev. 34-35 where *Nin-ug(!)-da* appears as the wife (*dam-bi-sal*) of Ninazu. Not less surprising is Harper's statement (*loc. cit.*) that Zamama is a goddess—notwithstanding the fact that he himself calls him (*loc. cit.* p. 107) "the great warrior, the chief son of Ekur."

HUGO RADAU.

CHICAGO, ILL., July, 1906.

CONGRÈS INTERNATIONAL DE PHILOSOPHIE. Rapports par Dr. Ed. Claparède.

Geneva: Kündig, 1905.

This is the secretary's report of the eleventh session of the International Congress of Philosophy, held at Geneva, September 4-8, 1904. The first part contains the rulings of the congress, the program, and lists of members, after which follow the leading addresses with portraits of the speakers. The frontispiece to the volume is the portrait of the venerable Honorary President of the Congress, M. Ernest Naville. Among the papers read at this memorable congress were: The Task of Logic, by Dr. W. Windelband; The Individual and Society, by M. Vilfredo Pareto; Neovitalism and Finality in Biology, by J. Reinke; Moral Personalities, by A. Borstel; The God of Aristotle, by Ch. Werner; "The World" of Descartes, by K. Jungmann; The Dominant Ideas in the Philosophy of Tolstoy, by Paul Biroukoff; An International Language, by Louis Couturat.

TRAITÉ DE PHYSIQUE. By O. D. Chwolson. Translated by E. Davaux. Paris: Hermann, 1906. Pp. 407.

E. Davaux, marine engineer, has translated Prof. O. D. Chwolson's work on Physics from the Russian, with the assistance of a German edition previously made under the supervision of the author. Messrs. E. and F. Cos-

serat, the former, professor of science at the University of Toulouse, the latter, chief engineer of the eastern railroads in France, have added notes on theoretical physics.

Professor Chwolson's prominence as a physicist is fully established among his colleagues. The present translation is an excellent one. It seems to be fully as reliable as the German version of the same work. The book is printed on good paper in clear type, and the many figures in the text are well done.

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DER URSPRUNG DER ISRAELITISCH-JÜDISCHEN ESCHATOLOGIE. Von Dr. Hugo Gressmann. Göttingen: Vandenhoeck & Ruprecht, 1905. Pp. viii, 378.

Prof. Hugo Gressmann of Kiel publishes in the present volume a series of books on the religion and literature of the Old and New Testaments, a careful enquiry concerning the origin of the Israelitish and Judæan eschatology, and we are glad to recognize the ability with which he has accomplished his task. His method is historical and critical. He follows Eichhorn mainly, but also Gunkel, Wellhausen, Giesebrecht, Smend and others, and handles the subject with independence and not without originality.

His work is divided into two parts,—the eschatology of misfortune, and the eschatology of salvation. These were the two thoughts which dominated the formation of a belief of the great events that were expected by the pious Jews. The Yahveh revelation was seen in earthquakes, storms, volcano-eruptions, in fire and thunder, in war and pestilence, etc., and the day of Yahveh was expected to come like a catastrophe upon the world.

But this eschatology of misfortune was offset by the hope of salvation, that God would not be forgetful of his promise of salvation to his chosen people. The idea of the Golden Age played an important part in this connection. A new covenant was expected, nature to be reclothed in an ideal shape and paradise restored. The central figure of the return of the Golden Age was the Messiah, later on identified with the Son of Man.

Professor Gressmann has not devoted a special chapter to Christianity, but we may add that the Jewish eschatology is the stepping stone between Judaism and Christianity, and for that reason the results in this field are of great importance. They are sure to throw light upon the problem of the origin of our own religion. We do not hesitate to recommend the book to the attention of Biblical scholars.

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LA PHILOSOPHIE DE CHARLES RENOUVIER. Introduction à l'étude du néo-criticisme. Par Gabriel Séailles, professeur à la Sorbonne. Paris: Alcan. 1905. Pp. iv, 400. Price, 7 fr. 50.

M. Séailles believes that in the history of French philosophy of the nineteenth century Charles Renouvier will hold a place of honor near his great compatriot Auguste Comte. Trusting in his critical method, careful to show the far-reaching importance of his principles, convinced, too, that error is the cause of evil, Renouvier is not slow to express what he believes to be the truth. Logic, psychology, natural philosophy, moral and political history—all of these he has touched upon and investigated.

M. Séailles intends his book to be an introduction to neo-criticism, a perfectly clear exposé of a doctrine difficult of comprehension. He has in-



sented a large number of quotations; he has gathered together a number of formulas, and has expressed himself throughout with force and precision. Criticism is subordinated to intelligence of thought and serves only to bring it into prominence. The author is especially interested in the exposition of Renouvier's early philosophy; convinced that upon this will rest his significance with posterity; and he does not emphasize the religious beliefs to which the great philosopher arrived late in life except to point out how they enervate and contradict the original theses of neo-criticism.

The work of M. Séailles will be a valuable guide to those who wish to be initiated into neo-criticism, and a useful resumé of the work of the founder of this celebrated system of philosophy.

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LE MORALISME DE KANT ET L'AMORALISME CONTEMPORAIN. Par *Alfred Fouillée*. Paris: Félix Alcan, 1905. Pp. 375.

Alfred Fouillée is one of the most forcible and fertile writers on philosophical subjects in modern France, and he has devoted his special attention to the moral and social problems of our own age. The present book is intended to complement his former labors of moral theories by pointing out the contrast which obtains since the rise of Nietzsche's philosophy between moralism and unmoralism,—the latter being that system of philosophy which looks upon morality as a fraud and as positively injurious to the development of the super-man, the representative of higher humanity.

Fouillée contrasts Kant's formal moralism to Nietzsche's unmoralism. He himself takes a stand which is sympathetic with Kant, although he accepts the views of the sage of Königsberg not without restrictions. The principle of unmorality shows itself in two ways, as a hankering after pleasure, and a hankering after power. Both of them are will-o'-the-wisps which should be discarded, while Kant's system remains in force, and Fouillée would only deem a proper adjustment to our modern acquisitions in science necessary. He says in conclusion:

"If nature be only the material mechanism, Kant would be right in saying that morality is superior to nature in so far as morality implies a law of finality which is different from the law of material causation; but if nature be the All, and comprise also ourselves, if it be the entirety of life and of thought, then morality with the intelligent finality which it involves is not superior to nature—then it is the higher nature. Nature is the great All and morality which the worshipers of pleasure and of power try in vain to suppress is a part of the All, but being that which is pregnant with things to come it is worth more than the whole in the sense of the Greek saying *πλεον ημισυ παντός*, i. e., 'the half is more than the whole.'"



